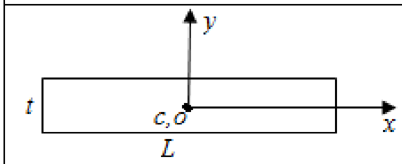
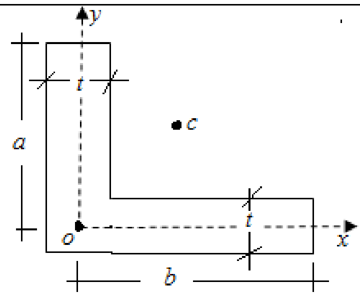
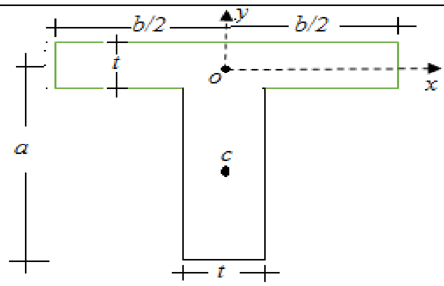
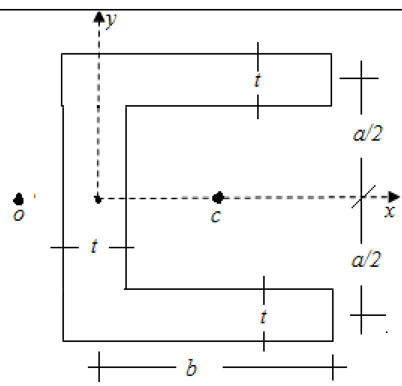
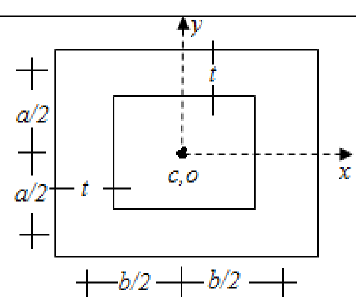
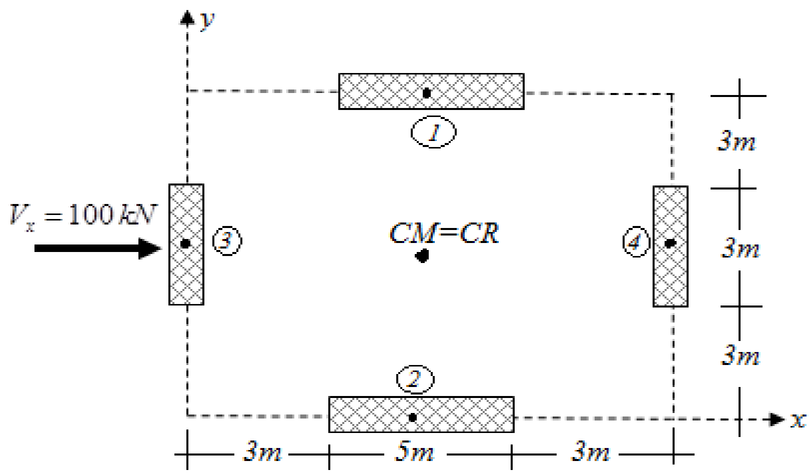
Table 5.1: Centroid (C), Shear Center (O) and Moment of Inertia (I) for d/t shape of shear walls

Shape of shear walls	Centroid	Shear Center	Moment of Inertia
	$\bar{x}=0$ $\bar{y}=0$	$x=0$ $y=0$	$\bar{I}_x = I_x = \frac{L\cdot t^3}{12}$ $\bar{I}_y = I_y = \frac{t\cdot L^3}{12}$
	$\bar{x}=\frac{b^2}{2(a+b)}$ $\bar{y}=\frac{a^2}{2(a+b)}$	$x=0$ $y=0$	$\bar{I}_x=\frac{a^3\cdot t(a+4b)}{12(a+b)}$ $\bar{I}_y=\frac{b^3\cdot t(b+4a)}{12(a+b)}$
	$\bar{x}=0$ $\bar{y}=\frac{a^2}{2(a+b)}$	$x=0$ $y=0$	$\bar{I}_x=\frac{a^3\cdot t}{12}\left[1+\frac{(3ab)}{(a+b)^2}\right]$ $\bar{I}_y=\frac{b^3\cdot t}{12}$
	$\bar{x}=\frac{b^2}{a+2b}$ $\bar{y}=0$	$x=\frac{3b^2}{a+6b}$ $y=0$	$\bar{I}_x=\frac{a^2\cdot t}{12}[a+6b]$ $\bar{I}_y=\frac{b^3\cdot t}{3}\left[\frac{2a+b}{a+2b}\right]$
	$\bar{x}=0$ $\bar{y}=0$	$x=0$ $y=0$	$\bar{I}_x=\frac{a^2\cdot t}{6}[a+3b]$ $\bar{I}_y=\frac{b^2\cdot t}{6}[b+3a]$

E.g. 5.1. (Symmetrical shear assembly)

The floor plan of a shear wall system shown below is subjected to a storey shear $V_x=100kN$. Distribute the storey shear to the shear walls. Use wall thickness, $t= 200\ mm$.



Sol<u>n</u>: - 1) Center of Stiffness (X_S, Y_S)

For a symmetrical shear assembly, center of mass and center of stiffness coincides.

$$CR = (X_S, Y_S) = (5.5, 4.5)$$

The floor plan only translates in the direction of V_x and there is no twisting effect on the plan.

2) Distribution of storey shear (V_{ix}, V_{iy})

$$V_{ix} = \frac{V_x \cdot I_{yi}}{\sum I_{yi}} \;;\; V_{iy} = \frac{V_y \cdot I_{xi}}{\sum I_{xi}}$$

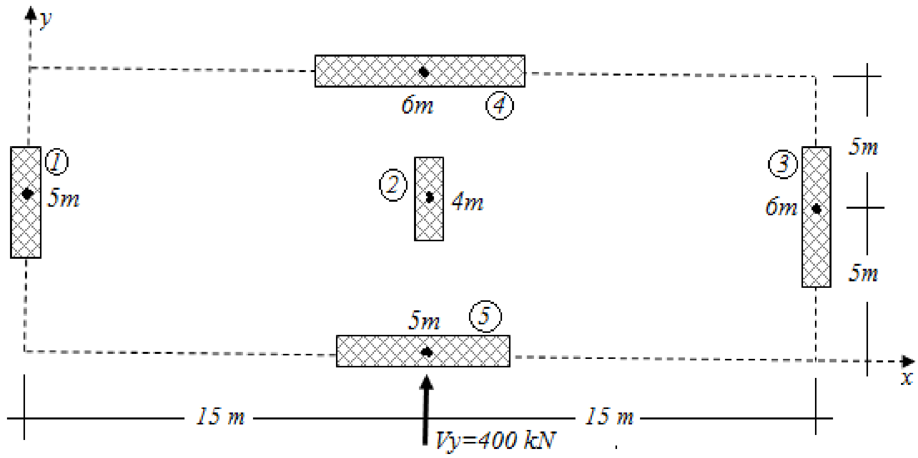
$$V_x = 100\,kN;\; V_y = 0$$

Shear wall	I_{xi}	I_{yi}	V_{ix}	V_{iy}
1	0	2.0833	50	0
2	0	2.0833	50	0
3	0.45	0	0	0
4	0.45	0	0	0
$\sum$	0.9	4.17		

Thus,
$$V_{1x} = 50\,kN;\; V_{2x} = 50\,kN\ and$$
$$V_{1y} = 0\;;V_{2y} = 0$$

E.g. 5.2. (Unsymmetrical shear assembly)

The figure below shows a floor plan of a shear wall multi storey structure. The horizontal shear in the storey under consideration is V_y*=400kN* acting along the center of the building. Determine the portion of lateral load resisted by each shear wall. Use thickness of wall *t= 300 mm.*



Sol<u>n</u>: 1) Center of Stiffness (X_S, Y_S)

$$x_s = \frac{\sum(I_{xi} \cdot x_i)}{\sum I_{xi}} \; ; \quad y_s = \frac{\sum(I_{yi} \cdot y_i)}{\sum I_{yi}} \; ; I_{xi} = \frac{bh^3}{12} \; ; I_{yi} = \frac{hb^3}{12}$$

x_i , y_i the distance of the shear center of wall *i* from origin of chosen coordinate system.

Shear wall	x_i	y_i	I_{xi}	I_{yi}	$I_{xi} \cdot x_i$	$I_{yi} \cdot y_i$
1	0	5	3.125	0	0	
2	15	5	1.6	0	24	0
3	30	5	5.4	0	162	0
4	15	10	0	5.4	0	54
5	15	0	0	3.125	0	0
$\sum$			10.125	8.525	186	54

$$x_s = \frac{\sum(I_{xi} \cdot x_i)}{\sum I_{xi}} = \frac{186}{10.125} = 18.37m$$

$$y_s = \frac{\sum(I_{yi} \cdot y_i)}{\sum I_{yi}} = \frac{54}{8.525} = 6.33m$$

2) Eccentricities (e_x and e_y) and Torsional Moment (T_s)

$$CR = (X_S, Y_S) = (18.37, 6.33)$$

$$CM = (x_m, y_m) = (15, 5)$$

$$e_x = x_s - x_m = 18.37 - 15 = 3.37m$$

$$e_y = y_s - y_m = 6.33 - 5 = 1.33m$$

$$T_s = V_y \cdot e_x = 400\, kN\, (3.37m) = 1348\, kNm$$

3) Distribution of the storey shear (Vi_x, Vi_y)

$$V_{iy} = \frac{V_y \cdot I_{xi}}{\sum I_{xi}} + \frac{T_s(I_{xi} \cdot \overline{x_i})}{\sum\left[I_{xi} \cdot \overline{x_i}^2 + I_{yi} \cdot \overline{y_i}^2\right]}$$

$$V_{ix} = \frac{V_x \cdot I_{yi}}{\sum I_{yi}} + \frac{T_s(I_{yi} \cdot \overline{y_i})}{\sum\left[I_{xi} \cdot \overline{x_i}^2 + I_{yi} \cdot \overline{y_i}^2\right]} = \frac{T_s(I_{yi} \cdot \overline{y_i})}{\sum\left[I_{xi} \cdot \overline{x_i}^2 + I_{yi} \cdot \overline{y_i}^2\right]} \qquad \text{Because, } V_x = 0$$

Where $\overline{x_i} = x_s - x_i = 18.37 - x_i$ $\qquad \overline{y_i} = y_s - y_i = 6.33 - y_i$

Shear wall	x_i	y_i	$\overline{x_i}$	$\overline{y_i}$	I_{xi}	I_{yi}	$I_{xi} \cdot \overline{x_i}^2$	$I_{yi} \cdot \overline{y_i}^2$
1	0	5	18.37	1.33	3.125	0	1054.55	0
2	15	5	3.37	1.33	1.6	0	18.17	0
3	30	5	-11.63	1.33	5.4	0	730.39	0
4	15	10	3.37	-3.67	0	5.4	0	72.73
5	15	0	3.37	6.33	0	3.125	0	125.22
$\sum$					10.125	8.525	1803.11	197.95

$$\sum\left[I_{xi} \cdot \overline{x_i}^2 + I_{yi} \cdot \overline{y_i}^2\right] = 1803.11 + 197.95 = 2001.06$$

$$V_{1y} = \frac{(400)(3.125)}{10.125} + \frac{(1348)(3.125)(18.37)}{2001.06} = 162.13\, kN$$

$$V_{2y} = \frac{(400)(1.6)}{10.125} + \frac{(1348)(1.6)(3.37)}{2001.06} = 66.84\, kN$$

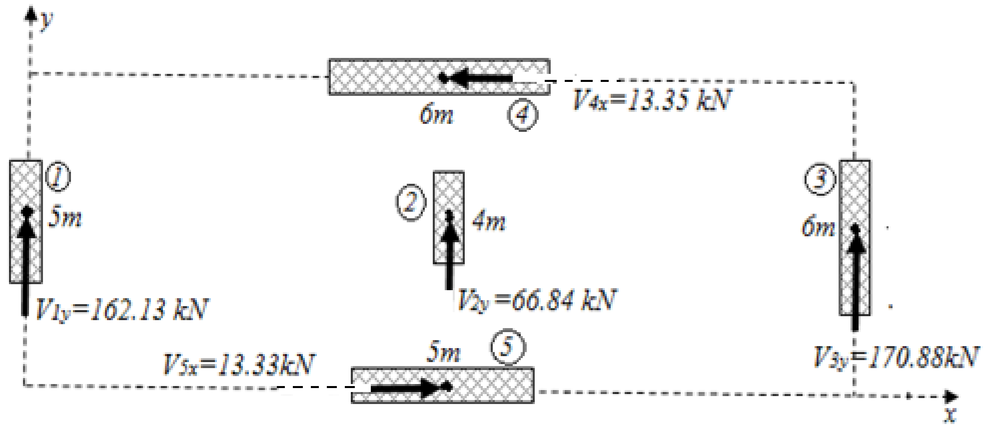
$$V_{3y} = \frac{(400)(5.4)}{10.125} + \frac{(1348)(-11.67)(5.4)}{2001.06} = 170.68\, kN$$

$$V_{4y} = V_{5y} = 0$$

$$V_{1x} = V_{2x} = V_{3x} = 0$$

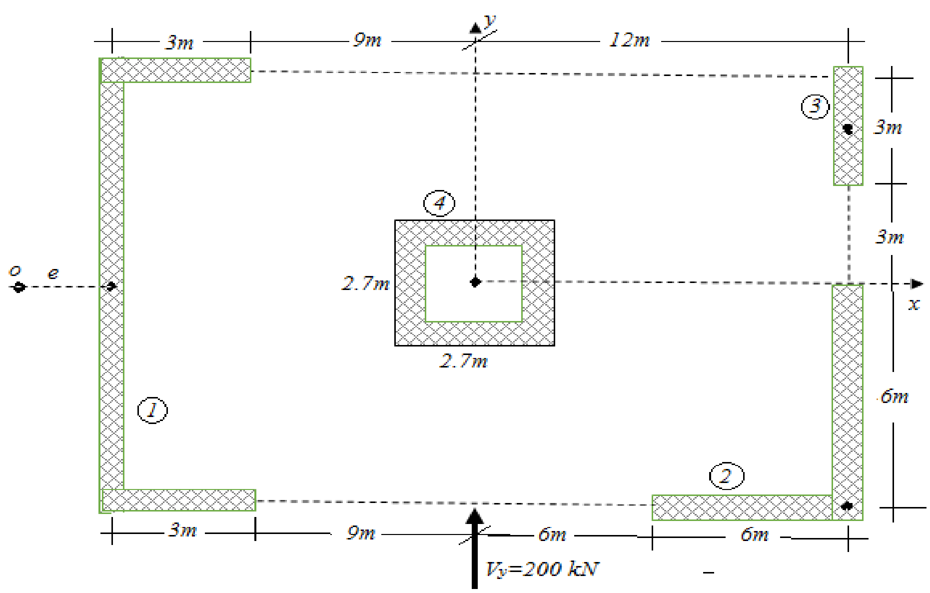
$$V_{4x} = \frac{(1348)(-3.67)(5.4)}{2001.06} = -13.35\ kN$$

$$V_{5x} = \frac{(1348)(6.33)(3.125)}{2001.06} = 13.33\ kN$$



E.g. 5.3. (Unsymmetrical shear assembly)

Distribute the storey shear V_y=200kN to various shear walls shown in the floor plan below. Use thickness of wall t= 200 mm.



Soln: 1) Center of Stiffness (X_S, Y_S)

$$x_s = \frac{\sum (I_{xi} \cdot x_i)}{\sum I_{xi}} \ ; \ \ y_s = \frac{\sum (I_{yi} \cdot y_i)}{\sum I_{yi}} \ ; \ e = \frac{3b^2}{a+6b} = \frac{3(3)^2}{12+6(3)} = 0.9m$$

Shear wall	x_i	y_i	I_{xi}	I_{yi}	$I_{xi}\cdot x_i$	$I_{yi}\cdot y_i$
1	-12.9	0	72	2.7	-928.8	0
2	12	-6	9	9	108	-54
3	12	4.5	0.497	0	5.96	0
4	0	0	2.08	2.08	0	0
$\sum$			83.58	13.78	-814.84	-54

$$x_s = \frac{\sum(I_{xi}\cdot x_i)}{\sum I_{xi}} = \frac{-814.84}{83.58} = -9.75m$$

$$y_s = \frac{\sum(I_{yi}\cdot y_i)}{\sum I_{yi}} = \frac{-54}{13.78} = -3.92\ m$$

2) Eccentricities (e_x and e_y) and Torsional Moment (T_s)

$CR = (X_S, Y_S) = (-9.75, -3.92)$; $CM = (x_m, y_m) = (0,0)$

$e_x = x_s - x_m = -9.75 - 0 = -9.75\ m\,; e_y = y_s - y_m = -3.92 - 0 = -3.92\ m$

$T_s = V_y \cdot e_x = 200\ kN\ (-9.75\ m) = -1950\ kNm$

3) Distribution the storey shear (Vi_x, Vi_y)

$$V_{iy} = \frac{V_y \cdot I_{xi}}{\sum I_{xi}} + \frac{T_s(I_{xi}\cdot \overline{x_i})}{\sum\left[I_{xi}\cdot \overline{x_i}^2 + I_{yi}\cdot \overline{y_i}^2\right]}$$

$$V_{ix} = \frac{V_x \cdot I_{yi}}{\sum I_{yi}} + \frac{T_s(I_{yi}\cdot \overline{y_i})}{\sum\left[I_{xi}\cdot \overline{x_i}^2 + I_{yi}\cdot \overline{y_i}^2\right]} = \frac{T_s(I_{yi}\cdot \overline{y_i})}{\sum\left[I_{xi}\cdot \overline{x_i}^2 + I_{yi}\cdot \overline{y_i}^2\right]} \qquad \text{Because } V_x = 0$$

Where $\overline{x_i} = x_s - x_i = -9.75 - x_i$; $\overline{y_i} = y_s - y_i = -3.92 - y_i$

Shear wall	x_i	y_i	$\overline{x_i}$	$\overline{y_i}$	I_{xi}	I_{yi}	$I_{xi}\cdot\overline{x_i}^2$	$I_{yi}\cdot\overline{y_i}^2$
1	-12.9	0	3.15	-3.92	72	2.7	714.42	41.49
2	12	-6	-21.75	2.08	9	9	4257.56	38.94
3	12	4.5	-21.75	-8.42	0.497	0	235.11	0
4	0	0	-9.75	-3.92	2.08	2.08	197.73	31.96
$\sum$					83.58	13.78	5404.82	112.39

$$\sum\left[I_{xi}\cdot \overline{x_i}^2 + I_{yi}\cdot \overline{y_i}^2\right] = 5404.82 + 112.39 = 5517.21$$

$$V_{1y} = \frac{(200)(72)}{83.58} + \frac{(-1950)(72)(3.15)}{5517.21} = 92.13\, kN$$

$$V_{2y} = \frac{(200)(9)}{83.58} + \frac{(-1950)(9)(-21.75)}{5517.21} = 90.73\, kN$$

$$V_{3y} = \frac{(200)(0.497)}{83.53} + \frac{(-1950)(0.497)(-21.75)}{5517.21} = 5.61\, kN$$

$$V_{4y} = \frac{(200)(2.08)}{83.53} + \frac{(-1950)(2.08)(-9.75)}{5517.21} = 12.15\, kN$$

$$V_{1x} = \frac{(-1950)(2.7)(-3.92)}{5517.21} = 3.74\, kN$$

$$V_{2x} = \frac{(-1950)(9)(2.08)}{5517.21} = -6.62\, kN$$

$$V_{3x} = \frac{(-1950)(0)(-8.42)}{5517.21} = 0$$

$$V_{4x} = \frac{(-1950)(2.7)(-3.92)}{5517.21} = 2.88\, kN$$