STEEL DESIGN - LRFD

FEBRUARY, 2003

Ref: AISC LRFD Specification, 1999
Ref: AISC LRFD Manual, 3rd Edition

Fluor, Aliso Viejo, CA

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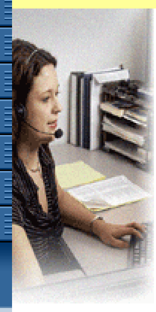

Highlights

- Bobbi Marstellar's Top Ten Skyscrapers
- Conceptual Solutions: Schematic Designs, Cost & Scheduling
- Free Downloads: Including AISC Codes & Specifications
- Engineering FAQs
- January Steel Quiz

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INTRODUCTION

LRFD MANUAL, 3RD EDITION

General

The design of structural steel buildings is based on information published in the American Institute of Steel Construction (AISC) Load And Resistance Factor Design (LRFD) Manual of Steel Construction.

The American Institute of Steel Construction (AISC) is made up of membership from:

- Steel producers
- Steel fabricators
- Design firms
- Universities

Part 1 - Dimensions And Properties

- Engineering properties listed in decimal inches, smallest number from rolling mills survey
- Fabrication dimensions listed to nearest $\frac{1}{16}in$, largest number from rolling mill survey

Part 2 – General Design Considerations

- The *Readers Digest* version of a textbook.

Part 3 – Design Of Tension Members

- Design aids to assist in selecting sections used as tension members.

Part 4 – Design Of Compression Members

- Design aids to assist in selecting sections used as columns.

Part 5 – Design Of Flexural Members

- Design aids to assist in selecting sections for flexural members.

Part 6 – Design Of Members Subject To Combined Loadings

- Design aids to assist in selecting sections for beam-columns.

Part 7 – Design Considerations For Bolts

- Design aids to assist in design of bolted connections.

Part 8 – Design Considerations For Welds

- Design aids to assist in design of welded connections.

Part 9 – Design Of Connection Elements

- Design aids to assist in design of connection elements (coped sections, block shear, etc.).

Part 10 – Design Of Simple Shear Connections

- Design aids to assist in design of simple shear connections.

Part 11 – Design Of Flexible Moment Connections

Part 12 – Design Of Fully Restrained (FR) moment Connections

Part 13 – Design Of Bracing Connections And Truss Connections

Part 14 – Design Of Beam Bearing Plates, Column Base Plates, Anchor Rods, And Column Splices

Part 15 – Design Of Hanger Connections, Bracket Plates And Crane-Rail connections

Part 16 - Specifications And Codes

Manual Page 16.1. Load And Resistance Factor Design Specification for Structural Steel Buildings

- Includes specification and commentary.

Manual Page 16.2. LRFD Sepcification for Steel Hollow Structural Sections

Manual Page 16.3. LRFD Specification for Single-Angle Members

Manual Page 16.4. Specification For Structural Joints Using ASTM A325 Or A490 Bolts

- Design information has been included in LRFD Specification.
- Includes requirements for “fully tensioned” bolts.

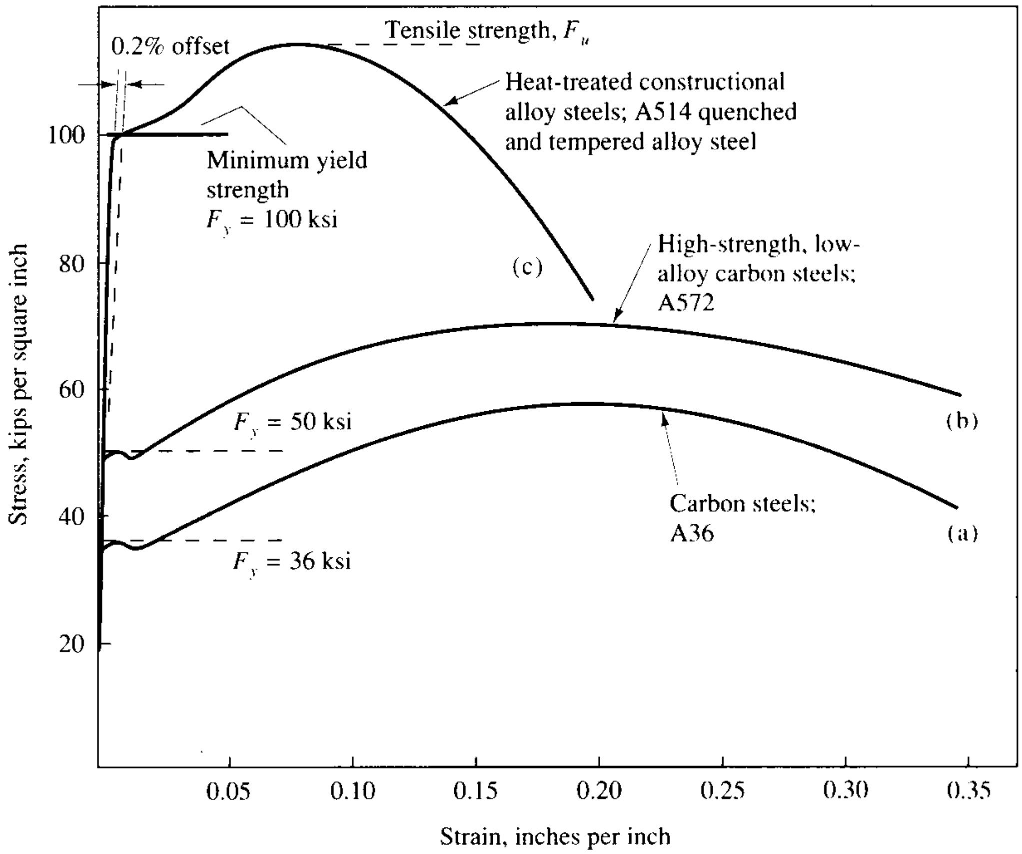
Manual Page 16.5. AISC Code of Standard Practice For Steel Buildings And Bridges

- Defines industry standard practice for fabrication and construction of structural steel.

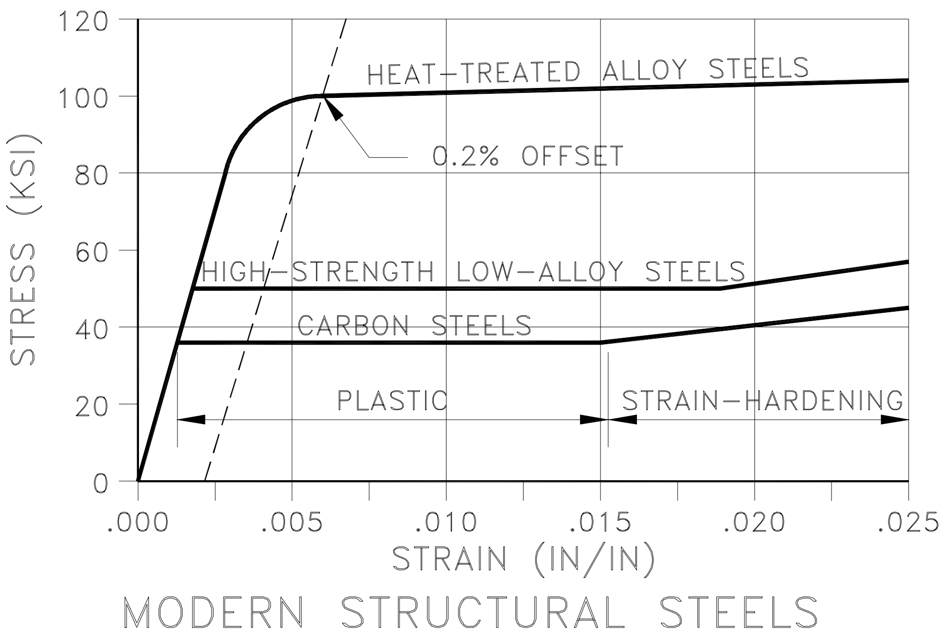
Part 17 - Miscellaneous Data And Mathematical Information

- Self-explanatory.

MATERIALS



Enlargement Of Plastic Range



Applicable ASTM Specifications

Manual Page 2-24. Table 2-1. Applicable ASTM Specifications For Various Structural Shapes

ASTM A36 was the most common structural carbon steel.

- $F_y = 36\,ksi$ {Manual 2-24}
- $F_u = 58\,ksi$ {Manual 2-24}
- Well-defined yield points.
- Course microstructure.
- Highly weldable.

ASTM A572, Grade 50 was the most common structural high-strength low-alloy steel.

- $F_y = 50\,ksi$ {Manual 2-24}
- $F_u = 65\,ksi$ {Manual 2-24}
- Weldable

ASTM A992 is currently used for W-shapes only.

- $F_y = 50\,ksi$ {Manual 2-24}
- $F_u = 65\,ksi$ {Manual 2-24}
- Weldable

Manual Page 2-25. Table 2-2. Applicable ASTM Specifications For Plates And Bars

ASTM A514 is the most common structural heat-treated alloy steel.

- F_y = 90 to 100 ksi {Manual 2-25}
- F_u = 100 to 130 ksi {Manual 2-25}
- Weldable with proper procedures
- 0.2% offset line is parallel to elastic curve, offset at the strain of 0.002 inch/inch.

Manual Page 2-26. Table 2-3. Applicable ASTM Specifications For Various Types Of Structural Fasteners

Manual Page 2-27. Table 2-4. Tensile Group Classification Of Structural Shapes

LOADS

ASCE 7-02

Minimum Design Loads for Buildings and Other Structures

- Provides minimum design loads for buildings and structures subject to building code requirements
- Loads are suitable for use with stresses and loads in design specifications such as ACI 318 and the LRFD Specification.
- Building Codes and Design Specifications refer to ASCE 7 for loads

Dead Loads

Dead loads are the gravity loads that do not vary with time in regards to position and weight. They are not moved once they are placed. They can be estimated with reasonable accuracy.

Live Loads

Live loads are the loads that vary with time in regards to magnitude and/or position. Floor and roof loads are the gravity loads produced by the use and occupancy of a building.

An estimated maximum value of a live load contains a much larger margin of error than an estimated dead load. These values are regarded as minimum live loads for building code purposes.

Snow Loads

Special type of roof live load; typical values range from 10 to 40 pounds per square foot.

- Usually determined or specified by local building department.
- Usually adjusted for roof slope and roof insulation.

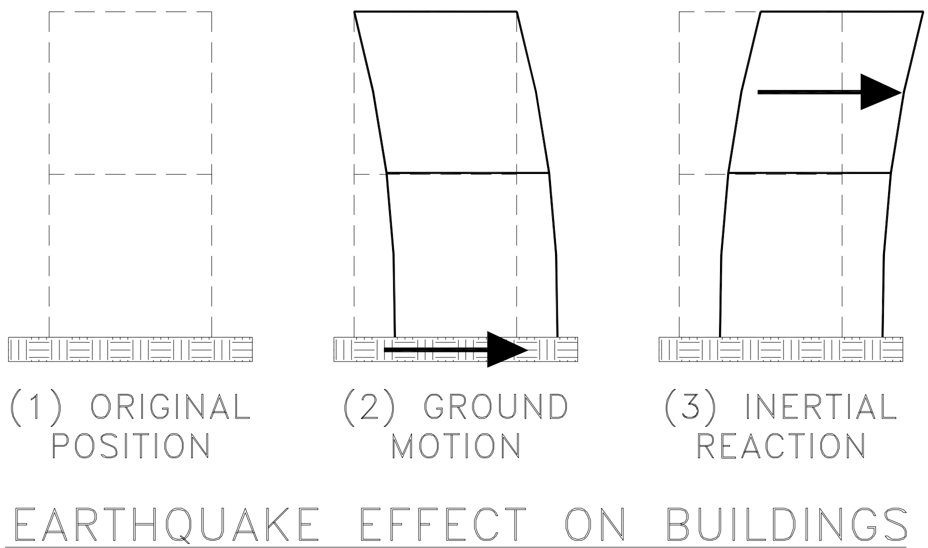
Rain Loads

Not as severe as snow loads.

- Ponding of rainwater can be a problem with flat roofs if roof drains are blocked or are undersized.

Earthquake Loads

During an earthquake, there is an input acceleration at the ground surface.



(1) The original building position is shown.

(2) The ground moves to the right, displacing the base of the building and leaving the top of the building in it's initial position. It's as if a "base shear" force was applied while the top of the building was held in place.

(3) The spring stiffness of the building frame brings the top of the building back over the base. Due to inertia effects, the top of the building overshoots the base of the building. This inertial effect can be conveniently looked at in terms of Newton's 2nd Law of Motion.

$$F = MA$$

Where:

- F = force, Kips
- M = mass, K-sec^2/ft
- A = acceleration, ft-sec^2

Earthquake loads on buildings must be accounted for. Design "base shear" equations take on the form of Newton's Law.

Wind Loads

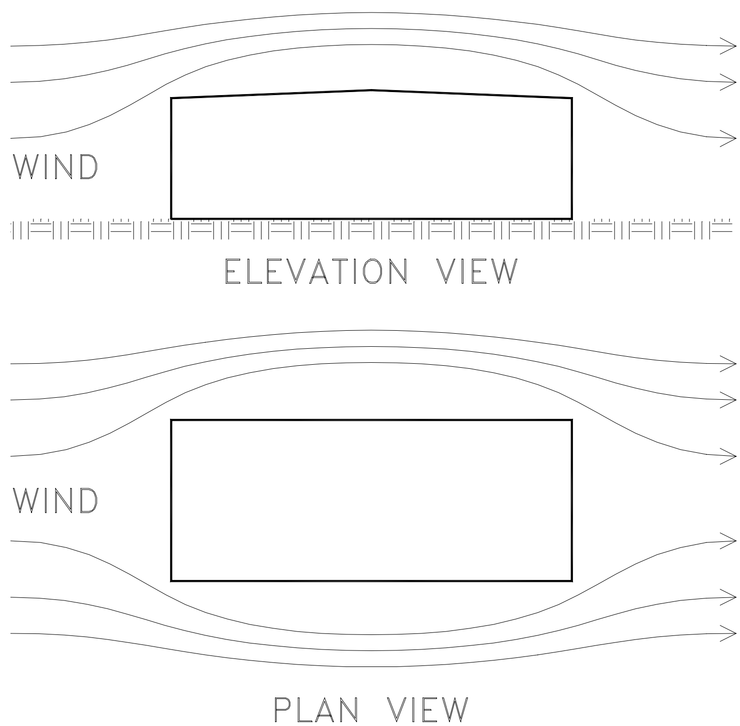
Wind pressures on the sides and roof of building must be accounted for.

$$Wind\ \mathrm{Pr}essure = q = \frac{\rho V^2}{2}$$

Where:

q = wind pressure, psf
ρ = mass of air, K-sec^2/ft
V = wind velocity, ft/sec

The wind pressures on structures usually act away from the building surface due to Bernoulli effects. As velocity increases, pressure decreases.



WIND EFFECT ON BUILDINGS

$$\frac{\rho V^2}{2} + P + \gamma H = \textit{Constant Energy}$$

Where:

P = atmospheric pressure, psf
γ = unit weight of air, #/ft^3
H = elevation of air stream, ft

ALLOWABLE STRESS DESIGN (ASD)

Basic Allowable Stress Formula

$$F_{allowable} = \frac{F_{failure}}{F.S.}$$

Factors Of Safety

Allowable steel stresses are usually expressed as some fraction of the material's specified yield strength (F_y) or ultimate tensile strength (F_u). The ASD method is characterized by the use of one factor of safety. This factor of safety concept is used in the determination of code allowable stresses. A limiting stress, either F_y,or F_u, is divided by a factor of safety.

$$F = \frac{F_y}{F.S.} or \frac{F_u}{F.S.}$$

Factors of safety are determined for different stress states based on the knowledge and experience of the code writers, with support from pertinent laboratory research.

Acceptance Relationships

The AISC ASD Specification uses lower case *f* for design stresses and upper case *F* for allowable stresses.

Demand ≤ Capacity

$$f \le F$$

If the actual stress were caused solely by dead and live loads:

$$f_a \le F_a$$

$$f_b \le F_b$$

$$f_v \le F_v$$

If the actual stresses were caused by a load combination including wind or seismic:

$$f_a \le 1.33F_a$$

$$f_b \le 1.33F_b$$

$$f_v \le 1.33F_v$$

LOAD AND RESISTANCE FACTOR DESIGN (LRFD)

Reliability

Reliability is a statistical concept. As used in this course, reliability is the estimated percentage of times the strength of the structure will equal or exceed the maximum loading applied to the structure during its estimated life.

The LRFD specification is based on a 99.7% Reliability that the strength of the structure will equal or exceed the maximum loading applied to the structure during its 50-year life.

- This means that for every 1000 buildings, only 3 are expected to have some members overloaded during their 50 year design lives.
- This does not mean that these 3 structures will collapse, just that some part of the structure will experience noticeable distress (yielding).

Limit States

Load and Resistance Factor Design (LRFD) is a method for designing steel structures, utilizing the concepts of probability based limit states design. A limit state is a condition that represents the limit of structural usefulness.

Serviceability Limit States define the functional requirements, such as vibration, deflection, and corrosion. Conformance with the Serviceability Limit States is mostly left to the judgment of the design engineer.

Strength Limit States define safety against the extreme loads during the intended life of the structure, such as yielding, fracture, or buckling. The LRFD Specification, like other structural codes, focuses on the strength limit states because of the overriding considerations of public safety. For an individual structural member, the Strength Limit State can be summarized as:

$$\text{Demand} \leq \text{Capacity}$$

$$U = \sum \gamma_i Q_i \leq \phi R_n \qquad \text{\{A5.3\}}$$

Where:

U = Required strength, kips or ft-kips

γ_i = Load factor, unitless

Q_i = Load effect, kips or ft-kips

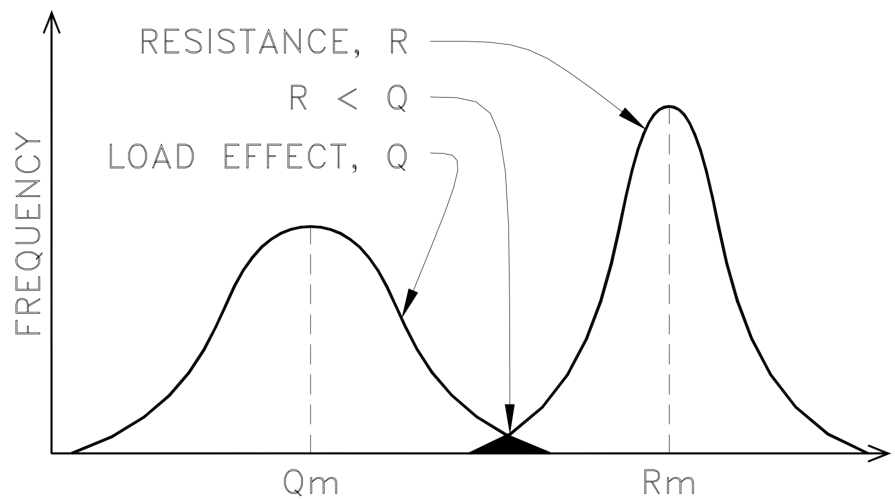
ϕ = Resistance factor, unitless

R_n = Nominal resistance, kips or ft-kips

ϕR_n = Design strength, kips or ft-kips

The frequency distribution of load effect and resistance can be shown graphically. The mean values (Q_m and R_m) are chosen such that there is a very small probability that $R < Q$ (indicated by the solid shading on the graph).

The 99.7% reliability is achieved based on the statistical analysis of many historical buildings. For each of these buildings the maximum *Q* and *R* are determined during its design life. Load factors (γ_l) and resistance factors (ϕ) are adjusted to give the desired reliability for each load combination and building element.



DISTRIBUTION OF LOAD EFFECTS AND RESISTANCE

Load Factors And Load Combinations

Manual Page 16.1-6: Load factors are materially independent and are based strictly on load statistics. Load factors and load combinations are defined in the applicable code or ASCE 7.

- $U = 1.4D$
- $U = 1.2D + 1.6L$
- $U = 1.2D + 0.5(L\, or\, 0.8W)$
- $U = 1.2D + 1.6W + 0.5L$
- $U = 1.2D + 1.0E + 0.5L$
- $U = 0.9D + 1.6W$
- $U = 0.9D + 1.0E$

Each load combination takes one load at its maximum level with the other loads at their probabilistic levels. The low probability of simultaneous load occurrence is accounted for by the load factors.

Nominal Strength

Resistances are given in the LRFD Specification for the various limit states. The primary resistances (R_n) are:

- Plastic Moment (M_p)
- Column Strength (P_{cr})
- Yield Stress (F_y)
- Yield Stress of Web (F_{yw})
- Yield Stress of Flange (F_{yf})
- Minimum Tensile Strength (F_u)

Resistance Factors

Cross-sectional properties indicated in the AISC Manuals are nominal values. The steel mills have tolerances (+ or -) for thicknesses and widths of flanges and webs. Similarly design strengths such as F_y and F_u are nominal values, usually specified minimums. To account for these **uncertainties**, resistance factors are used. The resistance factor is always less than equal or to 1.0 because there is always some **probability** that the actual resistance is less than the nominal resistance R_n.

The primary resistance factors are:

- $\phi_b = 0.90$ for bending
- $\phi_t = 0.90$ for yielding in tension member
- $\phi_v = 0.90$ for shear
- $\phi_c = 0.85$ for axial compression
- $\phi_t = 0.75$ for fracture in tension member
- $\phi_b = 0.65$ for bearing on most bolts
- $\phi_b = 0.60$ for bearing on concrete

EXAMPLE PROBLEM INTRO #1

GIVEN:

Building Column

$$D = 100\,K$$

$$L = 150\,K$$

$$L_r = 30K$$

$$S = 30K$$

$$R = 30\,K$$

$$W = 60\,K$$

$$E = 50K$$

REQUIRED: Determine required strength of column

SOLUTION:

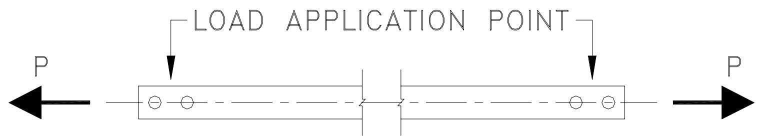
Load Combination	**Factored Load (K)**
$U = 1.4D$	$U = 1.4(100) = 140$
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or} R)$	$U = 1.2(100) + 1.6(150) + 0.5(30) = 375$
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or} R) + (0.5L \text{ or } 0.8W)$	$U = 1.2(100) + 1.6(30) + 0.5(150) = 243$
	$U = 1.2(100) + 1.6(30) + 0.8(60) = 216$
$U = 1.2D + 1.6W + 0.5L + 0.5(L_r \text{ or } S \text{ or} R)$	$U = 1.2(100) + 1.6(60) + 0.5(150) + 0.5(30)$ $= 306$
$U = 1.2D \pm 1.0E + 0.5L + 0.2S$	$U = 1.2(100) + 1.0(50) + 0.5(150) + 0.2(30)$ $= 251$
	$U = 1.2(100) - 1.0(50) + 0.5(150) + 0.2(30)$ $= 151$
$U = 0.9D \pm (1.6W \text{ or} 1.0E)$	$U = 0.9(100) + 1.6(60) = 186$
	$U = 0.9(100) - 1.6(60) = -6$
	$U = 0.9(100) + 1.0(50) = 140$
	$U = 0.9(100) - 1.0(50) = 40$

Required Strength = 375K Compression

Required Strength = 6K Tension

TENSION MEMBERS

Typical Tension Member



MEMBER SUBJECT TO TENSION LOAD

When a tensile force is applied through the centroidal axis of a member, the result is a uniform (average) tensile stress at each cross-section. The average stress of an axial loaded tension member is given by:

$$f = \frac{P}{A}$$

Where:

f = average stress across cross-section, ksi

P = axial load, Kips

A = cross-section area, in^2

This stress is exact, provided that the cross-section under consideration is not adjacent to the load application point, where stress distribution is not uniform. If the cross-sectional area varies along it's length, the stress is a function of the particular section under consideration.

POSSIBLE FAILURE MODES

AISC recognizes that a member loaded in axial tension can become unserviceable in several ways.

- The member could stretch so much that it no longer is useful to the structural system. For example a tension member in a truss that can no longer take any more load could cause the truss to fail.
- The member could locally stretch at bolt holes so much that loads couldn't transfer between the tension member and the rest of the structural system. This could also result in system failure.

GROSS AREA

Manual Page 16.1-10. Section B1. Gross Area: A_g is the total cross-sectional area of a tensile member taken along a transverse line where no holes are provided. If we apply enough tension load to cause the entire length of the section to reach yield, the total elongation of the member could be significant, rendering the member unserviceable.

NET AREA

Manual Page 16.1-62. Table J3.3. Nominal Hole Dimensions: When fabricating structural steel, standard bolt holes are usually punched $\frac{1}{16} in$ larger than the bolt diameter.

Manual Page 16.1-10. Section B2. Net Area: It is also assumed that another $\frac{1}{16} in$ of the surrounding area is damaged or destroyed. For design purposes, the bolt hole net area is equal to the bolt diameter plus $\frac{1}{8} in$. For non-staggered holes:

$$A_n = A_g - n\left(d + \frac{1}{8}\right)t$$

Where:

A_n = member net area, in^2
A_g = member gross area, in^2
n = number of bolt holes in cross-section taken transverse to tension force
d = bolt diameter, in
t = material thickness, in

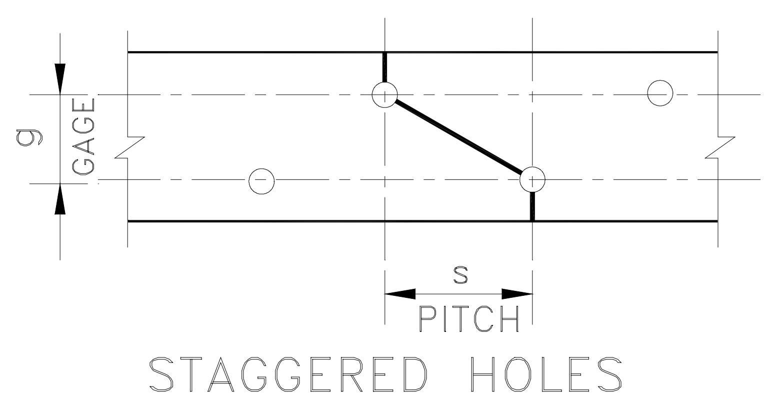
Staggered Fasteners

Manual Page 16-10. Section B2. Net Area: The effect of staggered holes is accounted for in the relationship:

$$A_n = A_g - n\left(d + \frac{1}{8}\right)t + \Sigma\left(\frac{s^2}{4g}\right)t$$

Where:

s = pitch, longitudinal center-to-center spacing of any two consecutive bolt holes, inch
g = gage, transverse center-to-center spacing between fastener gage lines, inch



Note that $\Sigma\left(\frac{s^2}{4g}\right)$ is an empirical relationship that adequately represents the effects of staggered holes. It is added to the net area to account for the diagonal distance between holes.

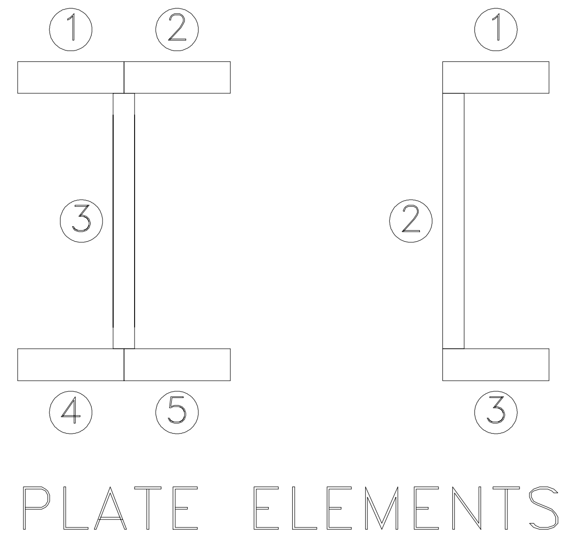
EFFECTIVE AREA

Effective Area, A_e, is referred to as Effective Net Area in previous AISC Specifications.

Manual Page 16.1-10. Section B3. Effective Area Of Tension Members: A structural shape consists of elements that make its shape.

For example:

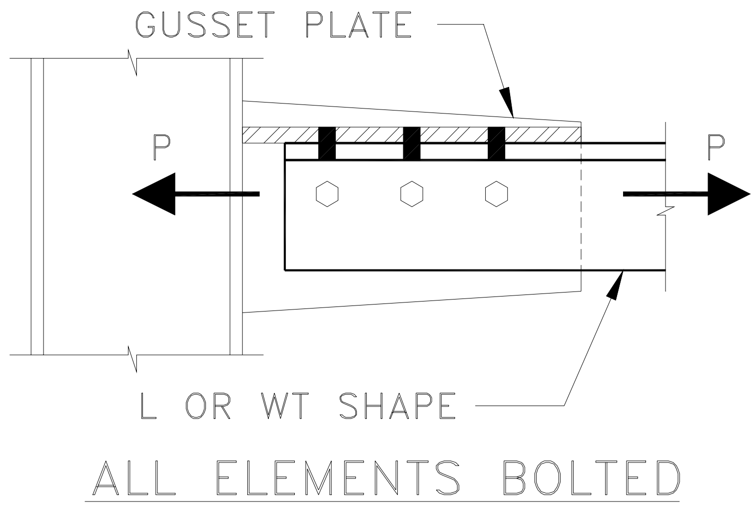
- A wide flange shape consists of five elements, two elements for each flange and one web.
- An angle consists of two elements, one element for each leg.



The concept of effective area addresses whether or not the transfer of tension loads from a structural shape to a fastener involves all or some of the elements of that shape.

If the distance to transfer the tension load between two members is short, the internal shear forces cannot be efficiently distributed from the entire cross-section (all of the elements) to the reduced cross-section (some of the elements) at the connection. This shear lag is accounted for by reducing the net area to an effective area.

All Elements Involved



(1) When a tension load is transmitted directly to each cross-sectional element by fasteners or welds, the effective area is equal to the net area:

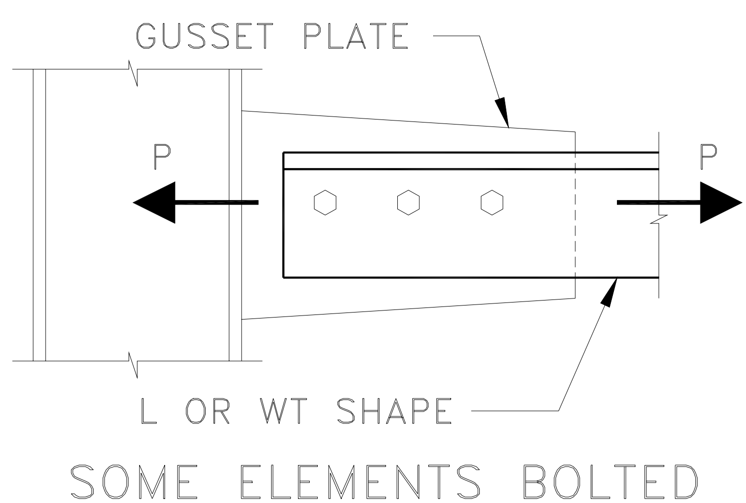
$$A_e = A_n$$

Where:

A_e = member effective area, in^2
A_n = member net area, in^2

(2) When a tension load is transmitted by fasteners or welds through some but not all of the cross-sectional elements of the member, the effective area shall be computed as follows:

Some Elements Involved



When the load is transmitted only by bolts:

$$A_e = A_n U \qquad \text{\{Eq. B3-1\}}$$

$$U = 1 - \left(\frac{\bar{x}}{L} \right) \le 0.9$$

Where:

U = reduction coefficient, unitless

$\bar{x}$ = connection eccentricity, inch

L = connection length in loading direction, inch

When the tension load is transmitted only by longitudinal welds to other than a plate member, or by longitudinal welds in combination with transverse welds:

$$A_e = A_g U \qquad \text{\{Eq. B3-2\}}$$

$$U = 1 - \left(\frac{\bar{x}}{L} \right) \le 0.9$$

When the tension load is transmitted only by transverse welds:

$$A_e = AU \qquad \text{\{Eq. B3-3\}}$$

$$U = 1.0$$

Where:

A = gross area of directly connected element, in^2

When the tension load is transmitted to a plate only by longitudinal welds along both edges of the plate:

$$A_e = A_g U \qquad \text{\{Eq. B3-4\}}$$

For $L \ge 2.0w$ $\qquad U = 1.00$

For $2.0w > L \ge 1.5w$ $\qquad U = 0.87$

For $1.5w > L \ge 1.0w$ $\qquad U = 0.75$

Where:

L = length of weld, in

w = plate width = distance between welds, inch

Manual Pages 16.1-178. Figure C-B3.1 Determination Of $\bar{x}$ For U: Figure indicates $\bar{x}$ for WF and C shapes. Note that when connecting to WF flanges, $\bar{x}$ is determined by treating it as a WT shape cut from it.

Manual Page 16.1-179. Figure C-B3.2. Staggered Holes: Figure indicates $\bar{x}$ for L shapes with staggered holes.

Manual Page 16.1-179. Figure C-B3.3 Longitudinal And Transverse Welds: Figure indicates "*L*" for various welded connections.

Estimates Of "U"

When the member size is not known, such as when selecting a tension member, estimate U as follows: {Manual 16.1-177}

- *U* = 0.90 for W, M, or S shapes with $b_f \le \frac{2}{3}d$, with 3 or more lines of fasteners to the flanges in the direction of loading
- *U* = 0.75 for W, M, or S shapes not included above, with 3 or more lines of fasteners to the flanges in the direction of loading
- *U* = 0.75 for all other cases

LIMITING SLENDERNESS RATIOS

Tension Members Other Than Rods

Manual Page 16.1-13. Section B7. Limiting Slenderness Ratios: For tension members other than rods, AISC prefers that $\frac{L}{r} < 300$.

Rods

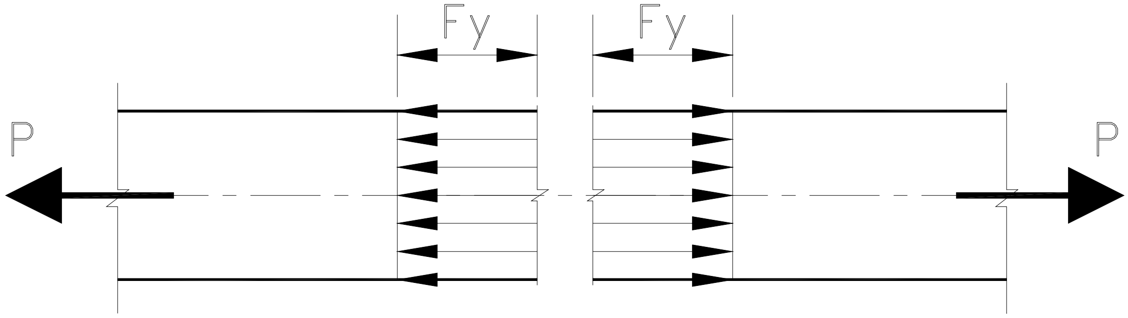
AISC does not provide limiting slenderness ratios for rods. Industry practice provides the following limitations:

$$d_{\min} = \frac{5}{8} in$$

$$\frac{L}{d} < 500$$

GROSS AREA YIELDING LIMIT STATE

The entire gross cross-sectional area reaches yield stress.



If we apply enough tension load to cause the entire length of the section to reach yield, the total elongation of the member could be significant, rendering the member unserviceable.

Specification Requirements

Manual Page 16.1-24. Section D1. Design Tensile Strength: The specification addresses this limit state by limiting the nominal tension member strength to the material strength times the member gross area.

$$P_u \le \phi_t P_n \qquad \{A5.3\}$$

$$\phi_t = 0.90$$

$$P_n = F_y A_g \qquad \{Eq.\ D1\text{-}1\}$$

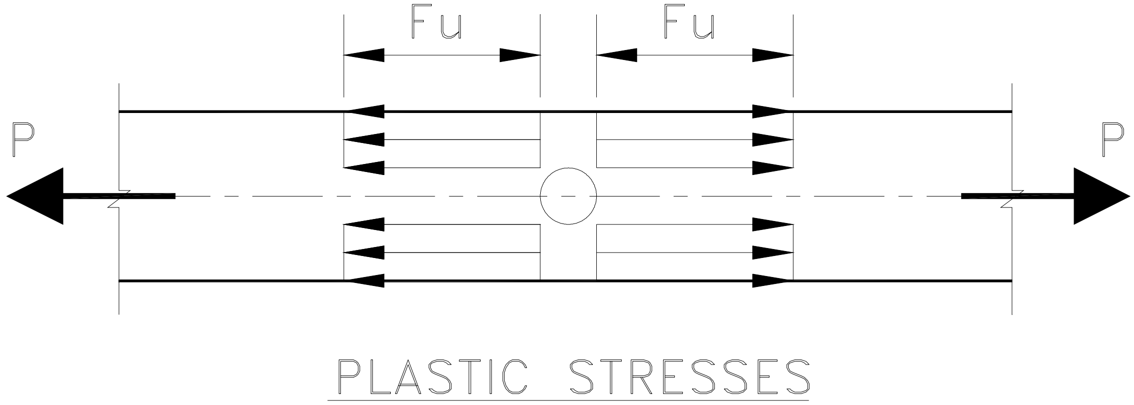
Where:

P_u = required tensile strength, Kips
ϕ_t = tension resistance factor, unitless
P_n = nominal axial strength, Kips
F_y = specified minimum yield stress, ksi
A_g = member gross area, in^2

ASD Sec. D1: The allowable stress F_t shall not exceed 0.60F_y on the gross area …

NET AREA FRACTURE LIMIT STATE

The entire net cross-sectional area fractures at bolt holes.



It is recognized that steel has a large capacity to take additional load without failure after the yield stress has been reached.

We consider this capacity when we look at local elongation at bolt holes, allowing the reduced cross-section at bolt holes to reach stresses between yield strength and ultimate tensile strength.

Specification Requirements

Manual Page 16.1-24: Section D1. Design Tensile Strength: The specification addresses this limit state by limiting the nominal tension member strength to the material ultimate tensile strength times the effective net area, A_e.

$$P_u \le \phi_t P_n \qquad \{A5.3\}$$

$$\phi_t = 0.75$$

$$P_n = F_u A_e \qquad \{Eq.\ D1\text{-}2\}$$

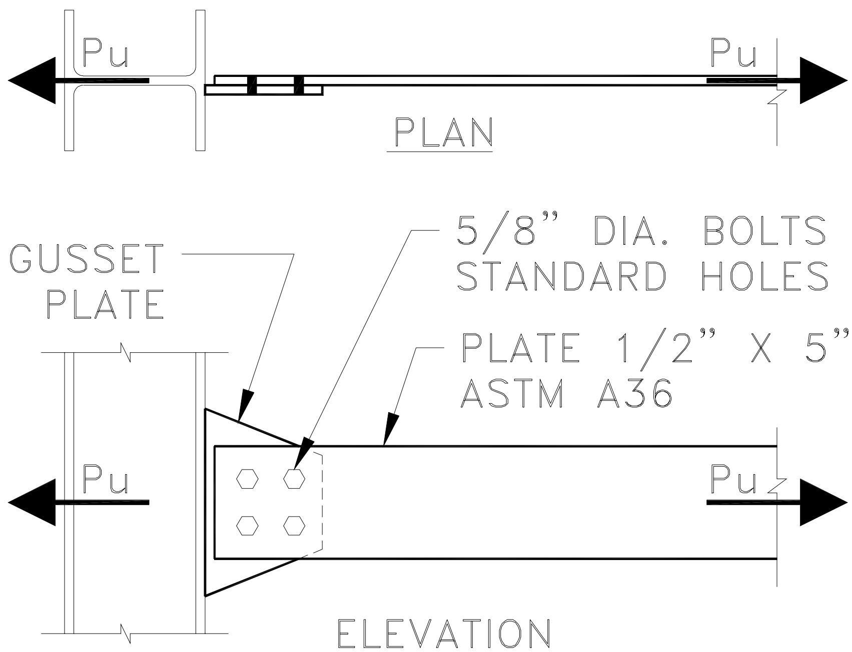
Where:

F_u = specified minimum tensile strength, ksi
A_e = member effective area, in^2

ASD Sec. D1: The allowable stress F_t shall not exceed … 0.50F_u on the effective net area.

EXAMPLE PROBLEM TENSION #1

GIVEN:



REQUIRED: Determine design strength of 1/2" plate

SOLUTION:

1) Design strength - tension on gross section

$$A_g = (0.5\,in)(5\,in) = 2.50\,in^2$$

$$\phi_t P_n = \phi_t F_y A_g = (0.90)(36\,ksi)(2.50\,in^2) = 81.0\,K$$

2) Design strength - tension on net section

$$A_n = 2.5in^2 - 2\left(\frac{5}{8}in + \frac{1}{8}in\right)\left(\frac{1}{2}in\right) = 1.75\,in^2$$

For plates, $A_e = A_n$

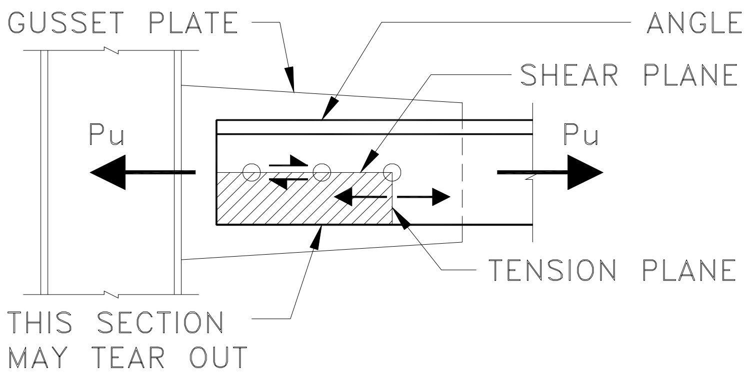
$$\phi_t P_n = \phi_t F_u A_e = (0.75)(58\,ksi)(1.75\,in^2) = 76.1\,K \quad \Leftarrow \text{lowest limit state}$$

Design Strength = 76.1K

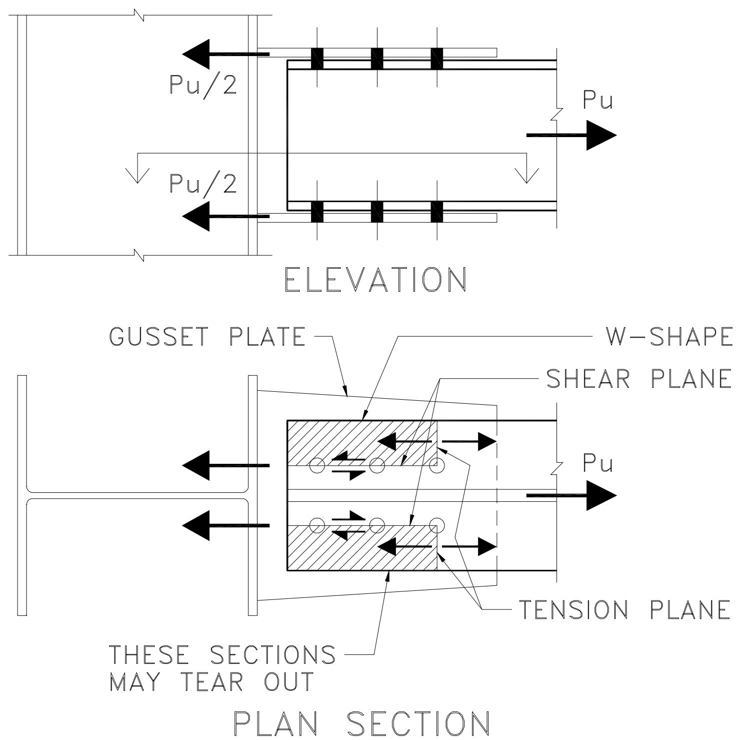
TEARING FAILURE AT BOLT HOLES

Block shear rupture is a phenomenon that didn't exist until the development of high strength bolts. With the increased capacity of the high strength bolts, it is now possible to cause a rupture failure in the base metal before the capacity of the bolts is reached. This block shear rupture must be checked in certain situations, such as the bolted connections of tension members.

Manual Page 16.1-67. Section J4. Design Rupture Strength



TENSION MEMBER BLOCK SHEAR – ANGLE



TENSION MEMBER BLOCK SHEAR – W–SHAPE

Shear Rupture Limit State

For shear failure surfaces in the direction of the applied load”

$$P_u \le \phi R_n \qquad \{A5.3\}$$

$$\phi = 0.75$$

$$R_n = 0.6 F_u A_{nv} \qquad \{Eq.\ J4\text{-}1\}$$

$$\boxed{F_v = 0.30 F_u} \qquad \boxed{\text{ASD Eq. J4-1}}$$

Where:

ϕ = resistance factor, unitless
R_n = nominal shear rupture strength, Kips
A_{nv} = member net shear area, in^2

Tension Rupture Limit State

For tension failure surfaces in the direction of the applied load

$$P_u \le \phi R_n \qquad \{A5.3\}$$

$$\phi = 0.75$$

$$R_n = F_u A_{nt} \qquad \{Eq.\ J4\text{-}2\}$$

Where:

A_{nt} = member net tension area, in^2

$$\boxed{F_t = 0.50 F_u} \qquad \boxed{\text{ASD Eq. J4-2}}$$

Block Shear Rupture Limit State

$$P_u \le \phi R_n \qquad \{A5.3\}$$

$$\phi = 0.75$$

When $F_u A_{nt} \ge 0.6 F_u A_{nv}$:

$$\phi R_n = \phi\left[0.6 F_y A_{gv} + F_u A_{nt}\right] \le \phi\left[0.6 F_u A_{nv} + F_u A_{nt}\right] \qquad \{Eq.\ J4\text{-}3a\}$$

When $F_u A_{nt} < 0.6 F_u A_{nv}$:

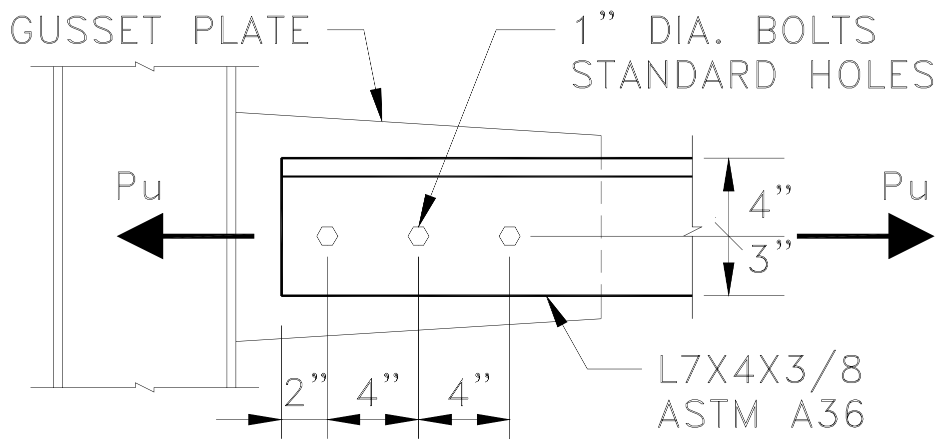
$$\phi R_n = \phi\left[0.6 F_u A_{nv} + F_y A_{gt}\right] \le \phi\left[0.6 F_u A_{nv} + F_u A_{nt}\right] \qquad \{Eq.\ J4\text{-}3b\}$$

Where:

A_{gv} = member gross shear area, in^2
A_{gt} = member gross tension area, in^2

EXAMPLE PROBLEM TENSION #2

GIVEN:



REQUIRED: Determine design strength of angle. Consider block shear.

SOLUTION:

1) Design strength - tension on gross section

$A_g = 4.00\,in^2$ {Manual 1-34}

$\phi_t P_n = \phi_t F_y A_g = (0.90)(36\,ksi)(4.00\,in^2) = 130\,K$

2) Design strength - tension on net section

$$A_n = 4.00\,in^2 - 1\left(1\,in + \frac{1}{8}in\right)\left(\frac{3}{8}in\right) = 3.58\,in^2$$

$\bar{x} = 0.861\,in$ {Manual 1-35}

$$U = 1 - \frac{\bar{x}}{L} = 1 - \left(\frac{0.861\,in}{8\,in}\right) = 0.89$$

$A_e = UA_n = (0.89)(3.58\,in^2) = 3.19\,in^2$

$\phi_t P_n = \phi_t F_u A_e = (0.75)(58\,ksi)(3.19\,in^2) = 139\,K$

3) Design strength - block shear

$$A_{gt} = \left(\frac{3}{8}in\right)(3\,in) = 1.12\,in^2$$

$$A_{nt} = 1.12\,in^2 - 0.5\left(1\,in + \frac{1}{8}in\right)\left(\frac{3}{8}in\right) = 0.91\,in^2$$

$$A_{gv} = \left(\frac{3}{8} in\right)(10\, in) = 3.75\, in^2$$

$$A_{nv} = 3.75\, in^2 - 2.5\left(1\, in + \frac{1}{8} in\right)\left(\frac{3}{8} in\right) = 2.70\, in^2$$

$$F_u A_{nt} = (58\, ksi)(0.91\, in^2) = 52.8\, K$$

$$0.6 F_u A_{nv} = 0.6(58\, ksi)(2.70\, in^2) = 94.0\, K$$

$F_u A_{nt} < 0.6 F_u A_{nv}$ Use Eq. J4-3b

$$F_y A_{gt} = (36\, ksi)(1.12\, in^2) = 40.3\, K$$

$$\phi R_n = \phi\left[0.6 F_u A_{nv} + F_y A_{gt}\right] \le \phi\left[0.6 F_u A_{nv} + F_u A_{nt}\right]$$ {Eq. J4-3b}

$$= (0.75)[94.0\, K + 40.3\, K] \le (0.75)[94.0\, K + 52.8\, K]$$

$$= 101\, K \le 110\, K$$

Summary

$$(\phi R_n)_{gross\,section} = 130\, K$$

$$(\phi R_n)_{net\,section} = 139\, K$$

$(\phi R_n)_{block\,shear} = 101\, K$ $\Leftarrow$ lowest limit state

Design Strength = 101K

DESIGN PROCEDURE

Design Considerations

$$P_u \le \phi_t P_n \qquad \text{\{A5.3\}}$$

- Compactness - Member no bigger than necessary.
- Dimensions that fit into the structure with reasonable relation to the dimensions of other members.
- Connections to as many parts of the tension member as possible, to minimize shear lag.

Gross Area Limit State

$$P_u \le \phi_t F_y A_g$$

$$A_{g(required)} \ge \frac{P_u}{0.90 F_y}$$

Effective Area Limit State

$$P_u \le \phi_t F_u A_e$$

$$P_u \le \phi_t F_u \left(1 - \frac{\bar{x}}{L}\right)\left(A_g - holes\right)$$

$$A_{g(required)} \ge \frac{L P_u}{0.75 F_u \left(L - \bar{x}\right)} + holes$$

Slenderness Ratio

Except For Rods

$$\frac{L}{r_{\min}} \le 300$$

$$L \le 300 r_{\min}$$

$$r_{\min} \ge \frac{L}{300}$$

Rods (Industry Practice)

$$\frac{L}{d_{\min}} \le 500$$

$$d_{\min} \ge \frac{L}{500}$$

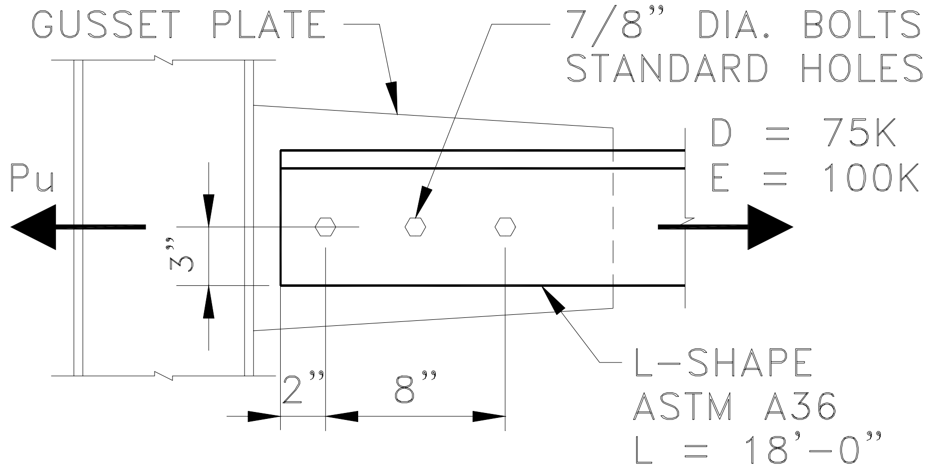
$$d_{\min} = \frac{5}{8} in$$

Block Shear Rupture Limit State

Check bolt (or weld) geometry for block shear rupture.

EXAMPLE PROBLEM TENSION #3

GIVEN:



REQUIRED: Select the lightest section. Consider block shear.

SOLUTION:

1) Required strength

$$P_u = 1.4D = 1.4(75\,K) = 105K$$

$$P_u = 1.2D + 1.6L = 1.2(75\,K) + 1.6(100\,K) = 190\,K$$ <u>governs</u>

2) Required gross area

$$A_{g(required)} \geq \frac{P_u}{\phi_t F_y} = \frac{190\,K}{0.90(36\,ksi)} = 5.86\,in^2$$

$$A_{g(required)} \geq \frac{LP_u}{0.75F_u\left(L - \bar{x}\right)} + holes$$

$$= \frac{8\,in(190\,K)}{0.75(58\,ksi)\left(8in - \bar{x}\right)} + 1\left(\frac{7}{8}in + \frac{1}{8}in\right)t = \frac{34.9}{\left(8 - \bar{x}\right)} + 1.00t\,in^2$$

3) Required slenderness

$$r_{\min} \geq \frac{18\,ft(12\,in/ft)}{300} = 0.72\,in$$

4) Select member

	PROVIDED				REQUIRED			
SIZE	Weight (#/ft)	A_g (in²)	r_z (in)	x (in)	A_g (in²)	A_g (in²)	r_z (in)	
L5x5x5/8	20.1	5.90	0.975	1.47	5.86	5.98 ng	0.72	ng
L6x4x5/8 (LLV)	19.8	5.83	0.860	1.03	5.86	5.63	0.72	ok

5) Check L6x4x5/8 (LLV) block shear design strength

$$A_{gt} = (0.625\,in)(3\,in) = 1.88\,in^2$$

$$A_{nt} = 1.88\,in^2 - (0.5)\left(\frac{7}{8}in + \frac{1}{8}in\right)\left(\frac{5}{8}in\right) = 1.57\,in^2$$

$$A_{gv} = (0.625\,in)(10\,in) = 6.25\,in^2$$

$$A_{nv} = 6.25\,in^2 - (2.5)\left(\frac{7}{8}in + \frac{1}{8}in\right)\left(\frac{5}{8}in\right) = 4.69\,in^2$$

$$F_u A_{nt} = (58\,ksi)(1.57\,in^2) = 91.1\,K$$

$$0.6F_u A_{nv} = 0.6(58\,ksi)(4.69\,in^2) = 163\,K$$

$F_u A_{nt} < 0.6F_u A_{nv}$ Use Eq. J4-3b

$$F_y A_{gt} = (36\,ksi)(1.88\,in^2) = 67.7\,K$$

$$\phi R_n = \phi\left[0.6F_u A_{nv} + F_y A_{gt}\right] \le \phi\left[0.6F_u A_{nv} + F_u A_{nt}\right] \quad \text{\{Eq. J4-3b\}}$$

$$= (0.75)[163\,K + 67.7\,K] \le (0.75)[163\,K + 91.1\,K]$$

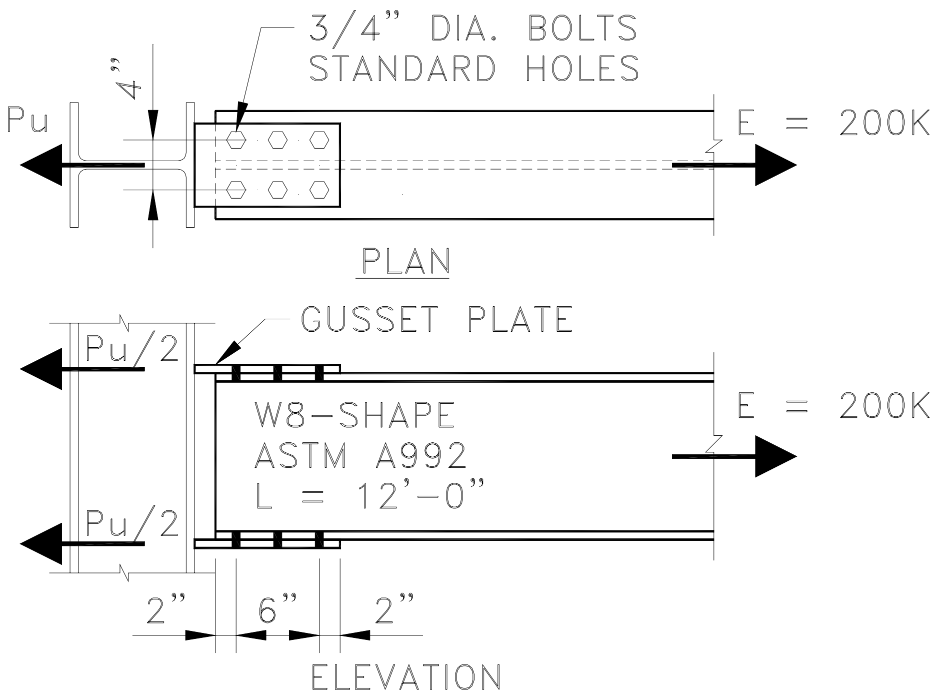
$$= 173\,K \le 191\,K$$

$\phi R_n = 191\,K > 190\,K = P_u$ ok

Select: L6x4x5/8 (LLV), increase transverse edge distance to 4 inch

EXAMPLE PROBLEM TENSION #4

GIVEN:



REQUIRED: Select the lightest section. Consider block shear rupture.

SOLUTION:

1) Required strength

$$P_u = 1.0(200\,K) = 200\,K$$

2) Required gross area

$$A_{g(required)} \geq \frac{P_u}{\phi_t F_y} = \frac{200\,K}{0.90(50\,ksi)} = 4.44\,in^2$$

$$A_{g(required)} \geq \frac{LP_u}{0.75F_u\left(L-\bar{x}\right)} + holes$$

$$= \frac{6\,in(200\,K)}{0.75(65\,ksi)\left(6\,in-\bar{x}\right)} + 4\left(\frac{3}{4}in+\frac{1}{8}in\right)t_f = \frac{24.6}{\left(6-\bar{x}\right)} + 3.5t_f\ in^2$$

3) Required slenderness

$$r_{\min} \geq \frac{L}{300} = \frac{12\,ft(12\,in/ft)}{300} = 0.48\,in$$

4) Select member

	PROVIDED				REQUIRED		
SIZE	t_f (in)	A_g (in^2)	r_y (in)	$\bar{x}$ (in)	A_g (in^2)	A_g (in^2)	r_y (in)
W8x18	0.33	5.26	1.23	0.83	4.44 ok	**5.92 ng**	0.48 ok
W8x21	0.40	6.16	1.26	0.83	4.44 ok	6.16 ok	0.48 ok

5) Check W8x21 block shear rupture design strength

$$A_{gv} = 4(0.40\,in)(8\,in) = 12.8\,in^2$$

$$A_{nv} = 4(0.40\,in)\left[8\ in - 2.5\left(\frac{3}{4}in + \frac{1}{8}in\right)\right] = 9.30\ in^2$$

$$A_{gt} = 4(0.40\,in)\left(\frac{5.27\,in - 4\,in}{2}\right) = 1.02\,in^2$$

$$A_{nt} = 4(0.40\,in)\left[0.635\ in - 0.5\left(\frac{3}{4}in + \frac{1}{8}in\right)\right] = 0.32\ in^2$$

$$F_u A_{nt} = (65\,ksi)(0.32\,in^2) = 20.8\,K$$

$$0.6F_u A_{nv} = 0.6(65\,ksi)(9.30\,in^2) = 363\,K$$

$F_u A_{nt} < 0.6 F_u A_{nv}$ Use Eq. J4-3b

$$\phi R_n = \phi\left[0.6F_u A_{nv} + F_y A_{gt}\right] \le \phi\left[0.6F_u A_{nv} + F_u A_{nt}\right] \qquad \text{\{ Eq. J4-3b \}}$$

$$= 0.75\left[0.6(65\,ksi)(9.30\,in^2) + (50\,ksi)(1.02\,in^2)\right] \le 0.75\left[363\,K + 20.8\,K\right]$$

$$= 310\,K \le 288\,K$$

$\phi R_n = 288\,K > 200\,K = P_u$ ok

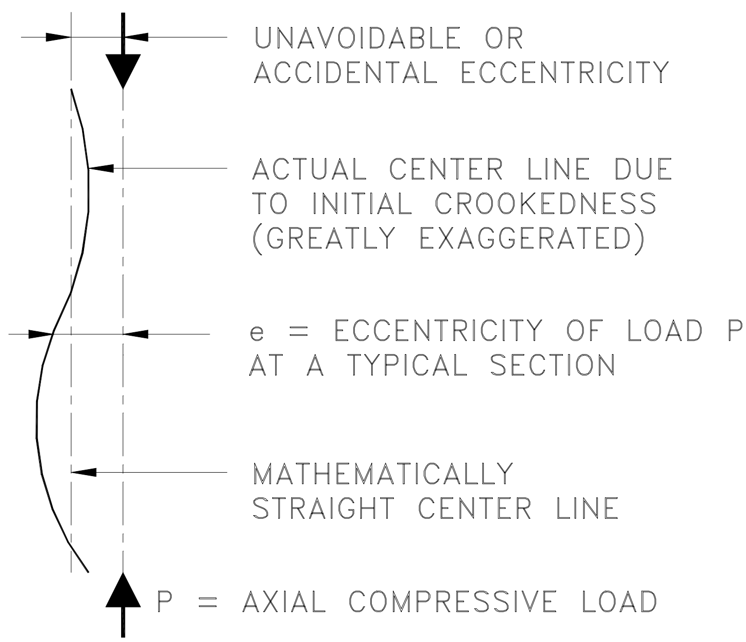
Select: W8x21

COMPRESSION MEMBERS

ECCENTRICITIES

Ideal concentrically loaded columns do not exist. All columns have accidental eccentricities due to:

- material imperfections
- types of end connections
- initial crookedness of column
- eccentric loads on columns
- residual stresses



COLUMN LOADING ECCENTRICITIES

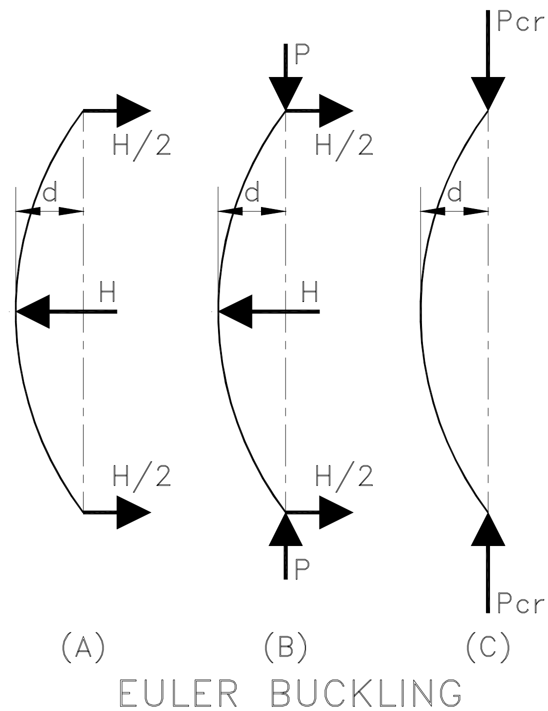
Application of load *P* at eccentricity, *e*, introduces a flexural stress.

- If *e* is small and member is short, the lateral deflection due to flexure is small and the flexural stress is insignificant.
- If the member is long, it is very flexible because deflection is proportional to the length cubed; even a small *e* can introduce significant flexural stresses.

EULER BUCKLING

Leonhard Euler was a Swiss Mathematician.

Critical Loads



(A) Long beam is mounted vertically and hinged at both ends so that it is free to bend in any direction. A central horizontal load *H* is applied, causing a midspan deflection of *d*.

(B) Axial load *P* is applied at each beam end.

(C) Axial load *P* is increased. Horizontal load *H* is decreased such that the midspan deflection "*d*" is maintained. The *P* when $H = 0$ is defined as the critical load, P_{cr}. In other words, P_{cr} is the critical load required to maintain the column in it's deflected position without any side thrust.

Any increase in axial load beyond P_{cr} increases the deflection (*d*) and the moment (*M*). Eventually, the column will buckle and fail elastically. The Euler buckling theory defines the critical load, P_e.

Euler Buckling Equations & Variations

Critical load formula: $$P_e = \frac{\pi^2 EI}{L^2}$$

Critical stress variation: $$F_e = \frac{P_e}{A} = \frac{\pi^2 EI}{L^2 A}$$

Noting that: $$r = \sqrt{\frac{I}{A}}$$

As used in LRFD: $$F_e = \frac{\pi^2 EI}{L^2\left(\frac{I}{r^2}\right)} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

Where:

- P_e = Euler buckling load, Kips
- E = modulus of elasticity, ksi
- I = member moment of inertia in plane of consideration, in^4
- L = member length, in
- F_e = Euler buckling stress, ksi
- A = member gross area, in^2
- r = member radius of gyration, in

This is the Euler Buckling Formula, the critical stress for a hinged-end column.

Comments On Euler Formula

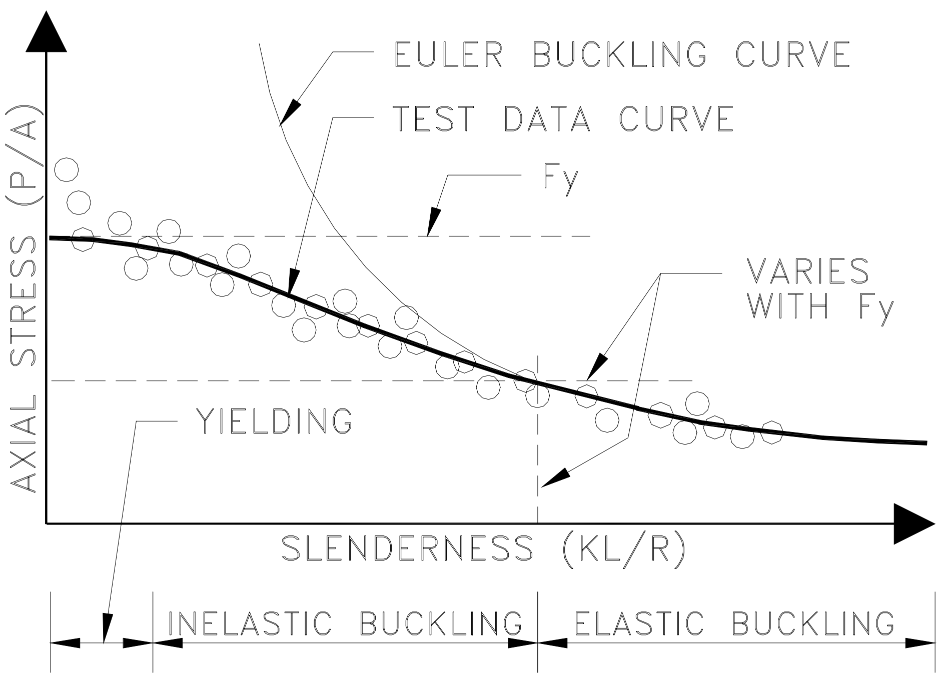
- A more mathematical derivation of the Euler Buckling Formula can be found in most Strength Of Materials textbooks.
- Every column has an x-axis and y-axis, each with its own I, r, and L. Every column will buckle about the axis with the highest $\frac{KL}{r}$ ratio.
- F_y and F_u have no effect on buckling tendency. A36, A992, and A514 steels all have the same modulus of elasticity (E) and will buckle at the same load for a given column size and support condition.
- If the Euler buckling stress exceeds the material yield stress, it is not applicable.
- Euler's formula defines the buckling load for a perfectly straight column.

FAILURE MODES OF COLUMNS

Conclusions based exclusively on Euler buckling theory are unconservative because it is based on the assumptions of elastic material behavior, and perfectly straight members, neither of which are true.

In reality, Euler buckling is an unattainable upper bounds limit state. This is contrary to most LRFD limit states that are defined conservatively, neglecting capacity beyond yield strains. Nevertheless, it is convenient to define and understand compression limit states in terms of Euler buckling.

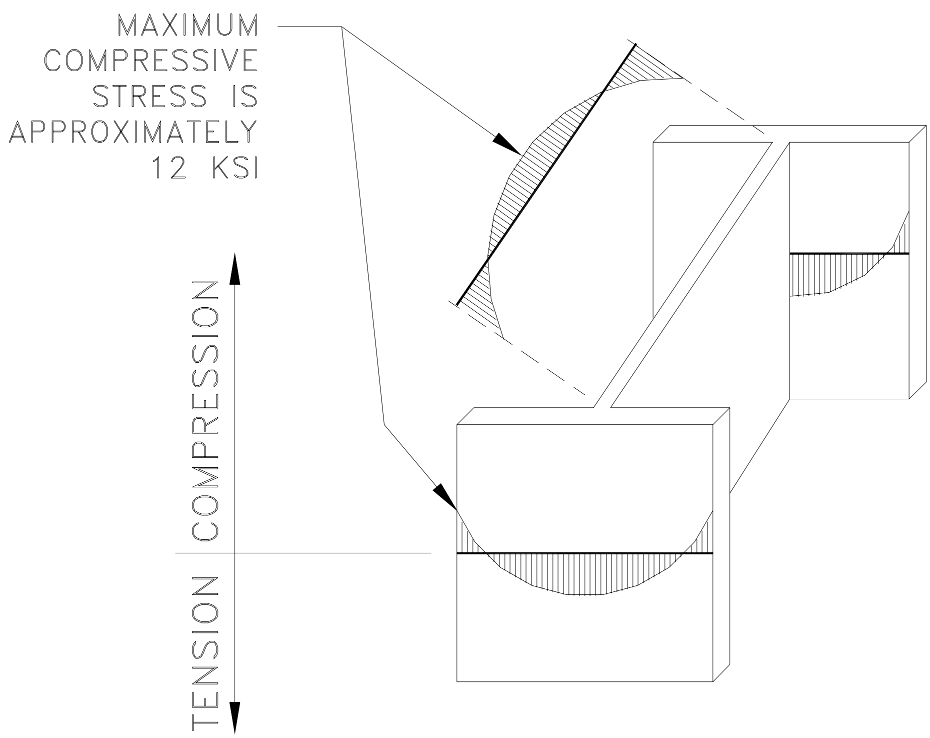
Test results indicate that many columns tend to fail before elastic flexural (Euler) buckling can be achieved.



FAILURE MODES OF COLUMNS

RESIDUAL STRESSES

Residual stresses are the stresses that remain in a member after it has been formed into a finished product.



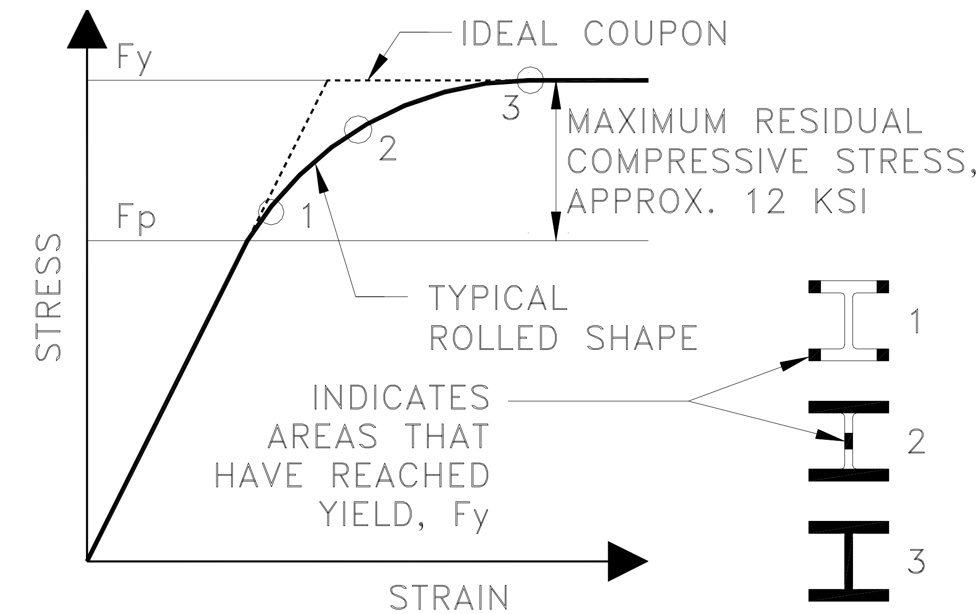
TYPICAL RESIDUAL STRESS PATTERN

Sources of residual stresses in structural steel include:

- Uneven cooling which occurs after hot rolling of structural shapes.
- The thicker flanges cool more slowly than the thinner webs.
- Flange tips have greater exposure to air and cool more quickly.
- Compression residual stresses exist in regions that cool the quickest.
- Tension residual stresses occur in the regions that cool the slowest.
- Cold bending or cambering during fabrication
- Punching of holes during fabrication
- Cutting during fabrication
- Welding during fabrication, a special case of uneven cooling

Rolled Shapes

All fibers on cross-section are not stressed at the same level. Residual stresses cause early yielding, followed by inelastic behavior.



RESIDUAL STRESSES IN ROLLED SHAPES

As a column's compression load is increased, the parts of the column with residual compressive stresses will reach the material yield stress and go into the plastic range of behavior. The stiffness of the column will be reduced and become a function of the part of the column cross-section that is still elastic. A column with residual stresses will behave as though it has a reduced cross-section. Intermediate length columns buckle during an inelastic state of stress due to residual stresses. The column must have an applied load sufficient to reach the inelastic range.

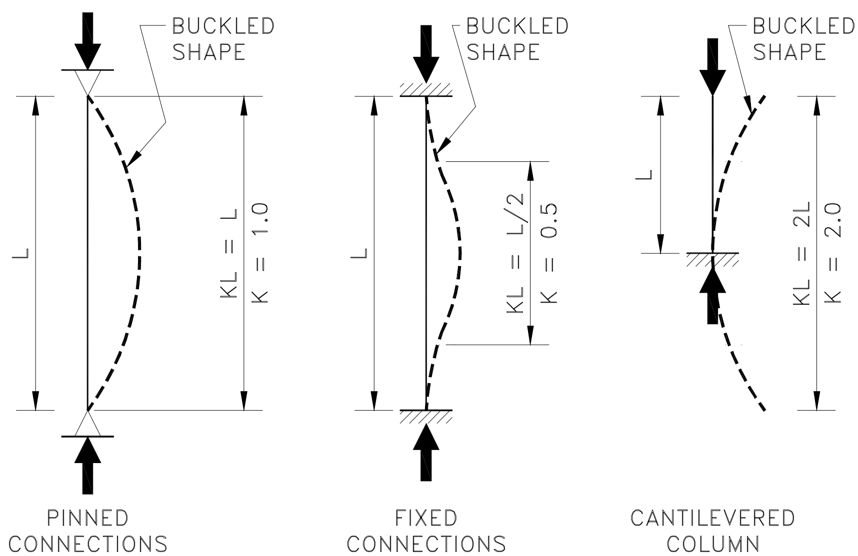
EFFECTIVE LENGTH

The concept of effective length is simply a mathematical method of replacing a given column with an equivalent pinned-end, braced column. In other words, the Euler buckling length is equal to *KL*.

Where:

K = effective length factor, unitless

L = column length between supports, in



COLUMN EFFECTIVE LENGTHS

The compression member's effective length can be defined as either:

- the distance between two consecutive inflection points, or
- the distance between two consecutive points of zero moment

LRFD Manual Page 16.1-189. Table C-C2.1. K Values For Columns

Case (a): fixed-fixed

- theoretical *K* = 0.5
- design *K* = 0.65

Case (d): pinned-pinned

- theoretical *K* = 1.0
- design *K* = 1.0

The effective length factor, *K*, is added to adjust the Euler buckling formula for different end conditions:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

LOCAL STABILITY

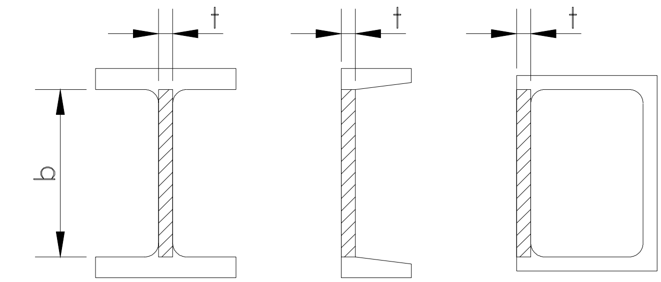
Rolled and built up shapes are made up of plate elements. The column strength of the overall section can only be achieved if the section plate elements do not buckle locally first.

Types Of Plate Elements

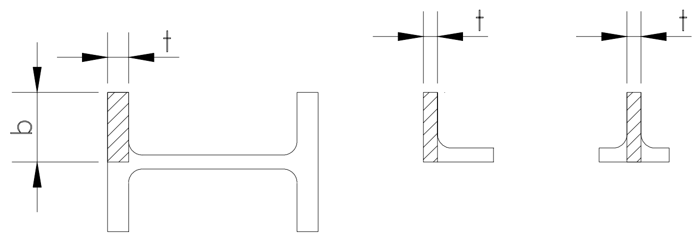
The section plate elements can be separated into two types, unstiffened and stiffened.

Unstiffened Elements are supported along only one edge parallel to the direction of load.

Stiffened Elements are supported along both edges parallel to the direction of load.



STIFFENED COMPRESSION ELEMENTS



UNSTIFFENED COMPRESSION ELEMENTS

- Examples are identified by $\frac{b}{t}$ or $\frac{h}{t_w}$ ratios.
- Note that for wide flange sections, the web is considered stiffened and the flanges are considered unstiffened.

Classification Of Cross-Section Shapes

- Compact Sections are stable at plastic strains, and are capable of developing the material yield stress across the entire member cross-section before buckling. Almost all commonly used W-shapes are compact for 50 ksi steels.
- Noncompact Sections become unstable with some amount of inelastic strains, and are capable of developing the material yield stress across some, but not all of the element cross-sections before buckling.
- Slender Compression Elements become unstable at elastic strains, and are not capable of developing the material yield stress anywhere in the member cross-section before buckling.

LRFD Requirements

Manual Page 16.1-12. Section B5.1. Classification Of Steel Sections: Steel sections are classified as compact, noncompact, or slender-element sections. For a section to qualify as compact, its flanges must be continuously connected to the web or webs and the width-thickness ratio of its compression elements must not exceed the limiting width-thickness ratios λ_p from Table B5.1.

- Compact sections: $\lambda < \lambda_p$

If the width-thickness ratio of one or more compression elements exceeds λ_p but does not exceed λ_r, the section is noncompact.

- Noncompact sections: $\lambda_p < \lambda < \lambda_r$

If the width-thickness ratio of any element exceeds λ_r from Table B5.1, the section is referred to as a slender-element compression section.

- Slender compression elements: $\lambda_r < \lambda$

Manual Page 16.1-14. Table B5.1. Limiting Width-Thickness Ratios For Compression Elements

Table is sorted by stiffened elements and unstiffened elements.

Element descriptions are listed. The width-thickness ratio (λ) is defined for each element, such as $\frac{b}{t}$ or $\frac{h}{t_w}$.

The values for λ_p and λ_r are defined for each width-thickness ratio, usually in terms of the material yield strength (F_y) and modulus of elasticity (E).

Manual Page 16.1-183. Figure C-B5.1. Selected Examples Of Table B5.1 Requirements: The most commonly used values for W-shapes are:

Flange axial compression: $\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$ $\boxed{ASD: \frac{95}{\sqrt{F_y}}}$

Web axial compression: $\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$ $\boxed{ASD: \frac{253}{\sqrt{F_y}}}$

Note that λ_p is undefined for axial compression. The LRFD specification does not distinguish between compact or noncompact elements for columns.

EXAMPLE PROBLEM COLUMN #1

GIVEN: W6x15, ASTM A992

REQUIRED: Investigate column for local stability.

SOLUTION:

1) Flanges - axial compression

$$\lambda = \frac{b_f}{2t_f} = 11.5$$ {Manual 1-23}

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29000\,ksi}{50\,ksi}} = 13.5$$

$\lambda < \lambda_r$ flange is either compact or noncompact

2) Web - axial compression

$$\lambda = \frac{h}{t_w} = 21.2$$ {Manual 1-23}

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29000\,ksi}{50\,ksi}} = 35.9$$

$\lambda < \lambda_r$ web is either compact or noncompact

Column Is Not A Slender Compression Element

ELASTIC FLEXURAL BUCKLING LIMIT STATE

Column Slenderness Parameter

For mathematical convenience, define new term λ_c:

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}}$$

$$\boxed{ASD: C_c = \sqrt{\frac{2\pi^2 E}{F_y}}}$$

Where:

λ_c = column slenderness parameter, unitless
K = effective length factor, unitless
L = column length between supports, inch
r = member radius of gyration, inch
F_y = specified yield stress, ksi
E = modulus of elasticity, ksi

The Euler buckling formula can be defined in terms of the column slenderness parameter.

$$F_e = \frac{F_y}{\lambda_c^2}$$

Out-Of-Straight Columns

The Euler buckling stress is an upper bounds stress based on an perfectly straight column. Statistical studies have considered the actual out-of-straightness of rolled sections used as columns. A mean-centered out-of-straightness value was found to be $\frac{ColumnLength}{1500}$. The Euler buckling formula is modified by a factor of 0.877 to account for this average out-of-straightness value.

$$F_e = \left(\frac{0.877}{\lambda_c^2}\right)F_y$$

Test results indicate that <u>long columns</u> tend to fail in elastic flexural buckling. The critical stress at which elastic flexural buckling occurs is best predicted by the classic Euler buckling formula for pinned-end columns, modified for mean-centered out-of-straightness.

Test results indicate that column behavior is correctly predicted by the <u>modified</u> Euler buckling formula when $\lambda_c > 1.5$.

Specification Requirements

Manual Page 16.1-27. Chapter E. Columns And Other compression Members

- Chapter E applies to compact and noncompact prismatic members subject to axial compression through the centroidal axis.
- Refer to Chapter H for members subject to combined axial compression and flexure.
- Refer to Appendix B5 for slender compression elements.
- Refer to Appendix F3 for tapered members.

$$\phi_c = 0.85$$

$$P_n = A_g F_{cr} \qquad \text{\{Eq. E2-1\}}$$

Where:

P_u = design compression strength, Kips
ϕR_n = design strength. Kips
ϕ_c = compression resistance factor, unitless
P_n = nominal compression strength, Kips
A_g = member gross area, in^2
F_{cr} = critical compression stress, ksi

For long columns (λ_c > 1.5), the critical buckling stress F_{cr} is now defined as:

$$F_{cr} = \left(\frac{0.877}{\lambda_c^{\ 2}}\right) F_y \qquad \text{\{Eq. E2-3\}}$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} \qquad \text{\{Eq. E2-4\}}$$

$$\boxed{If\ \frac{KL}{r} > C_c \Rightarrow F_a = \frac{12\pi^2 E}{23\left(KL/r\right)^2}} \qquad \boxed{\text{ASD Eq. E2-2}}$$

MAXIMUM SLENDERNESS RATIOS

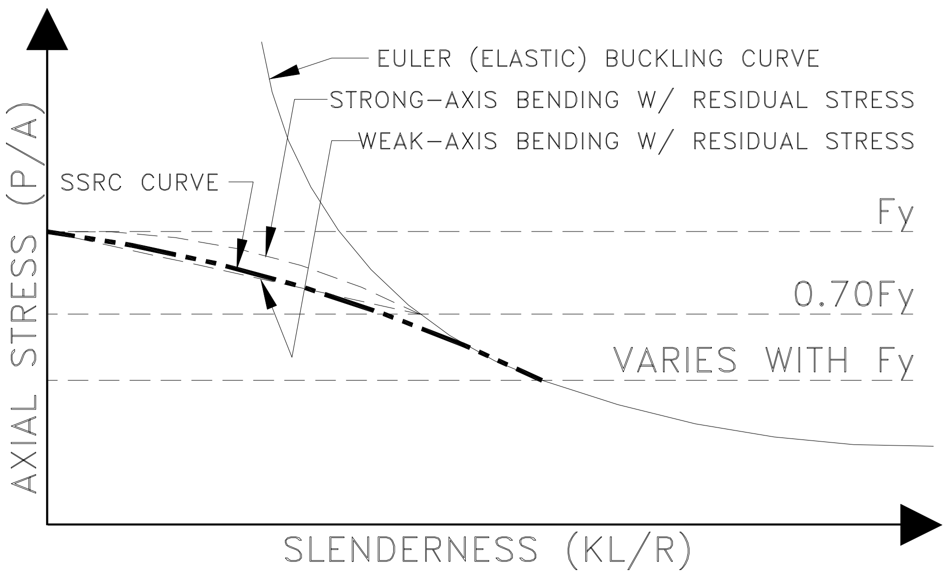
Manual Page 16.1-13. Section B7. Limiting Slenderness Ratios: AISC prefers that compression members do not have a slenderness ratio that exceeds 200.

- AISC does not say that you can't exceed 200, it just won't provide you any design aids if you do.
- If $KL/r > 200$, column will fail by elastic flexural buckling limit state. Determine design strength from equation E2-3.

INELASTIC FLEXURAL BUCKLING LIMIT STATE

SSRC Strength Curve

The Structural Stability Research Council (SSRC) performed inelastic analyses on several column configurations.



SSRC INELASTIC ANALYSES

Analyses accounting for residual stress tend to yield parabolic curves such as:

$$F_{cr} = F_y\left[1-\left(\frac{F_r}{\pi^2 E}\right)\left(\frac{F_y - F_r}{F_y}\right)\left(\frac{KL}{r}\right)^2\right]$$

Where:

F_r = residual stress, ksi

In terms of λ_c and with $F_r \approx 0.50F_y$;

$$F_{cr} = F_y\left[1-0.25\lambda_c{}^2\right]$$

Modifying by a factor of 0.877 to account for average out-of-straightness and simplifying:

$$F_{cr} = 0.658^{\lambda_c{}^2}F_y$$

Test results indicate that <u>short and intermediate columns</u> tend to buckle inelastically. Inelastic buckling is affected by residual stresses.

Specification Requirements

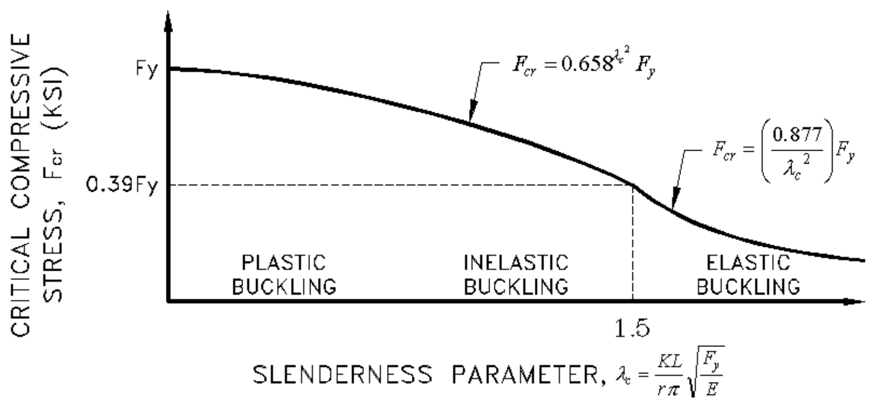
$$\phi_c = 0.85$$

$$P_n = A_g F_{cr} \qquad \text{\{Eq. E2-1\}}$$

$$F_{cr} = 0.658^{\lambda_c{}^2} F_y \qquad \text{\{Eq. E2-2\}}$$

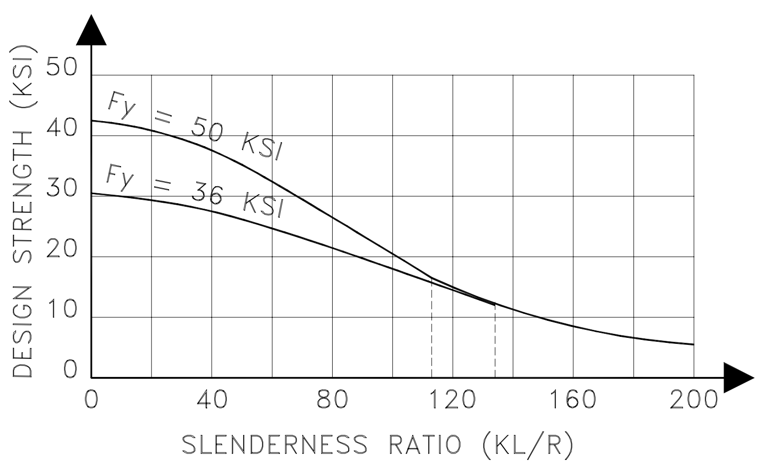
$$\boxed{If\ \frac{KL}{r} < C_c \Rightarrow F_a = \frac{\left[1 - \frac{\left(KL/r\right)^2}{2C_c{}^2}\right]F_y}{\frac{5}{3} + \frac{3\left(KL/r\right)}{8C_c} - \frac{\left(KL/r\right)^3}{8C_c{}^3}}} \qquad \boxed{\text{ASD Eq. E2-1}}$$

FLEXURAL BUCKLING SUMMARY



FLEXURAL BUCKLING LIMIT STATE

Design Column Strengths



DESIGN COLUMN STRENGTHS

TABLES FOR COMPRESSION MEMBERS

Column Stress Tables

An examination of Equations E2-2 and E2-3 indicates that they are a function of only $\frac{KL}{r}$, F_y and E.

- Solutions to the equation can easily by tabulated for common ranges of $\frac{KL}{r}$ and common values of F_y.
- AISC has tabulated values for $\frac{KL}{r} \le 200$ and $F_y = 36\,ksi$ or $F_y = 50\,ksi$ for use in analysis and design of compression members.

Manual Page 16.1-143. Table 3-36. Design Stress For Compression Members Of 36 KSI Specified Yield Stress, $\phi_c = 0.85$

- Tabulated values for ϕF_{cr} for $F_y = 36\,ksi$
- Includes both equations E2-2 and E2-3

Manual Page 16.1-145. Table 3-50. Design Stress For Compression Members Of 50 KSI Specified Yield Stress, $\phi_c = 0.85$

- Tabulated values for ϕF_{cr} for $F_y = 50\,ksi$
- Notice the gray shading.

Manual Page 16.1-147. Table 4. Values Of $\frac{\phi_c F_{cr}}{F_y}$ For Determining Design Stress For Compression Members For Steel Of Any Yield Stress, $\phi_c = 0.85$

- Can be used for any value of F_y.
- Notice that is uses λ_c, not $\frac{KL}{r}$.

Column Load Tables

It is possible to calculate the design strength for common column sizes, reasonable effective lengths, and common material strengths. AISC has done this.

The Manual presents column load tables for W, WT, Pipe, Tube, and Double Angles. Must use specification formulae for other shapes.

<u>Manual Page 4-21. W Shapes, Design Strength In Axial Compression, ϕ_c = 0.85, Kips</u>

Tables are presented for $F_y = 50\,ksi$ only. Must use specification formulae or Manual Table 4 for other stresses.

- Tables include $\frac{KL}{r} \leq 200$.
- Tables assume that weak-axis buckling will govern the column design and are calculated based on K_yL_y
- If *KL* is not the same for both axes, table may not indicate the governing allowable load.
- Ratio of $\frac{r_x}{r_y}$ in tables may be used to determine allowable loads when x-axis buckling governs the column design.
- The ratio $\frac{r_x}{r_y}$ will be covered in the design examples.

EXAMPLE PROBLEM COLUMN #2

GIVEN:

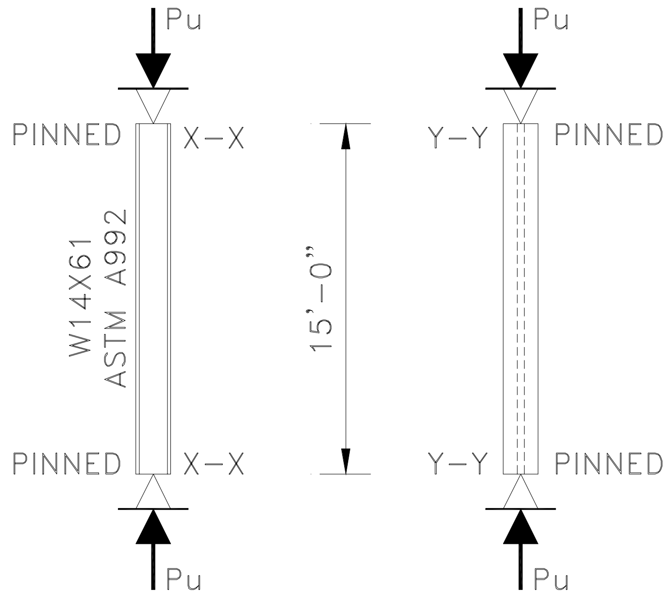
REQUIRED: Determine column design strength using:

a) specification equations

b) column stress tables

c) column load tables

SOLUTION:



1) Boundary conditions

K = 1.0 for pinned-pinned column

$$(KL)_y = (KL)_{max} = 1.0(15\,ft) = 15\,ft$$

2a) Using specification equations

$$\lambda_c = \frac{KL}{\pi r}\sqrt{\frac{F_y}{E}} = \frac{1.0(15\,ft)(12\,in/ft)}{\pi(2.45\,in)}\sqrt{\frac{50\,ksi}{29000\,ksi}} = 0.97$$ {Eq. E2-4}

$\lambda_c \le 1.5$ Use Eq. E2-2

$$F_{cr} = \left(0.658^{\lambda_c^2}\right)F_y = \left(0.658^{0.97^2}\right)(50\,ksi) = 33.7\,ksi$$ {Eq. E2-2}

$$\phi_c P_n = \phi_c F_{cr} A_g = 0.85(33.7\,ksi)(17.9\,in^2) = 513K$$

2b) Using column stress tables

$$\left(\frac{Kl}{r}\right)_y = \frac{1.0(15\,ft)(12\,in/ft)}{2.45\,in} = 73.5$$

$$\phi_c P_n = 28.6\,ksi$$ {Manual 16.1-145}

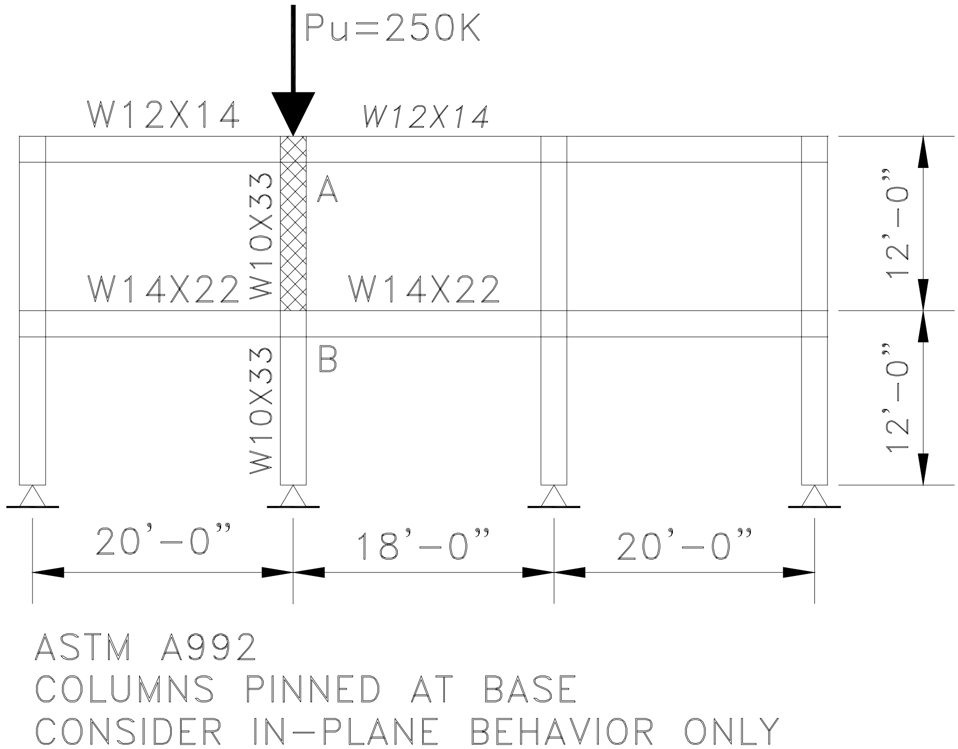
$$\phi_c P_n = \phi_c F_{cr} A_g = (28.6\,ksi)(17.9\,in^2) = 512\,K$$

2c) Using column load tables

$$\phi P_n = 513\,K$$ {Manual 4-23}

EXAMPLE PROBLEM COLUMN #3

GIVEN:



REQUIRED: Determine effective length factor of Member AB assuming

a) elastic column behavior

b) inelastic column behavior

SOLUTION:

1) Elastic column behavior

$$G_A = \frac{\sum\left(\frac{I_c}{L_c}\right)}{\sum\left(\frac{I_b}{L_b}\right)} = \frac{\left(\frac{170in^4}{12\,ft}\right)}{\left(\frac{88.6in^4}{20\,ft}\right)+\left(\frac{88.6in^4}{18\,ft}\right)} = 1.52$$

$$G_B = \frac{\sum\left(\frac{I_c}{L_c}\right)}{\sum\left(\frac{I_b}{L_b}\right)} = \frac{2\left(\frac{170in^4}{12\,ft}\right)}{\left(\frac{199in^4}{20\,ft}\right)+\left(\frac{199in^4}{18\,ft}\right)} = 1.35$$

K_x = 1.43 {Manual 16.1-192}

Assuming Elastic Behavior, K_x = 1.43

2) Inelastic column behavior

$$\frac{P_u}{A} = \frac{250\,K}{9.71\,in^2} = 25.7\,ksi$$

SRF = 0.83 {Manual 4-20}

$$G_A = \frac{\sum\left(\frac{I_c}{L_c}\right)}{\sum\left(\frac{I_b}{L_b}\right)} = \frac{\left(\frac{170in^4}{12\,ft}\right)0.83}{\left(\frac{88.6in^4}{20\,ft}\right)+\left(\frac{88.6in^4}{18\,ft}\right)} = 1.26$$

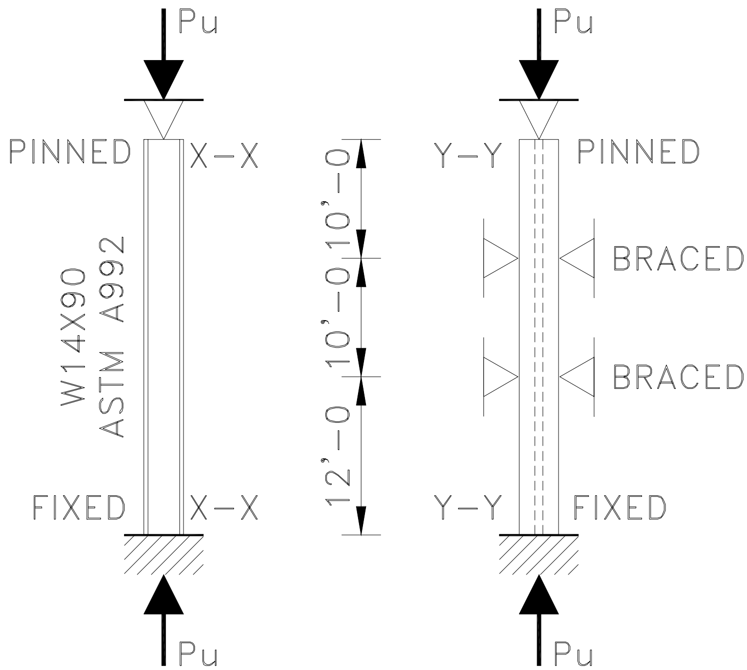
$$G_B = \frac{\sum\left(\frac{I_c}{L_c}\right)}{\sum\left(\frac{I_b}{L_b}\right)} = \frac{2\left(\frac{170in^4}{12\,ft}\right)0.83}{\left(\frac{199in^4}{20\,ft}\right)+\left(\frac{199in^4}{18\,ft}\right)} = 1.12$$

K_x = 1.37 {Manual 16.1-192}

Assuming Inelastic Behavior, K_x = 1.37

EXAMPLE PROBLEM COLUMN #4

GIVEN:



REQUIRED: Determine design strength. Use column load tables

SOLUTION:

1) W14x90 properties

A_g = 26.5 in^2

r_y = 3.70"

r_x = 6.14"

$r_x / r_y = 1.66$ {Manual 4-23}

2) Design strength - compression

Top two segments: $(KL)_y$ = 1.0(10 ft) = 10.0 ft {Manual 16.1-189}

Bottom segment: $(KL)_y$ = 0.8(12 ft) = 9.6 ft {Manual 16.1-189}

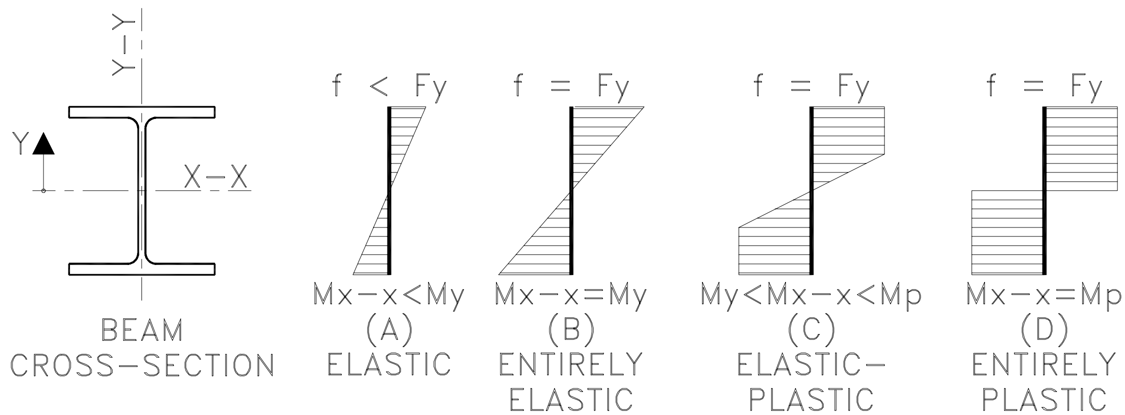
Entire column: $(KL)_{y(equiv)} = \dfrac{0.8(32\ ft)}{1.66} = 15.4\ ft$ <u>governs</u> {Manual 16.1-189}

$\phi_c P_n = 938\,K$ by interpolation {Manual 4-23}

Design Strength = 938 K

FLEXURAL MEMBERS

STRESS DISTRIBUTION AT DIFFERENT STAGES OF LOADING



STRESS DISTRIBUTION AT DIFFERENT LOADING STAGES

(A) Section is fully elastic.

$$M_{x-x} < M_y$$

(B) Section is fully elastic. Extreme fibers are at yield.

$$M_{x-x} = M_y$$

ASD Limit

(C) Outer fibers in plastic range. Inner fibers in elastic range.

$$M_y < M_{x-x} < M_p$$

(D) Section is fully plastic.

$$M_{x-x} = M_p$$

Where:

M_{x-x} = internal resisting bending moment about x-axis, in-K

$$M_{x-x} = \int_A fy\, dA$$

M_y = internal resisting bending moment about x-axis when extreme fiber has reached yield, in-K

M_p = internal resisting bending moment about x-axis when all fibers have reached yield, in-K

f = fiber stress, ksi

y = distance from neutral axis, in

dA = b dy = change in area with respect to distance y, in^2

FLEXURAL DESIGN

Manual Page 16.1-31. Section F1. Design For Flexure: the nominal flexural strength, M_n, is the lowest value of the following limit states:

- Yielding (Plastic Buckling)
- Flange Local Buckling (FLB)
- Web Local Buckling (WLB)
- Lateral Torsional Buckling (LTB)

The applicable specification section depends on both the flexural limit state and the local buckling limiting ratios.

	Compact Section $\lambda < \lambda_p$	Noncompact Section $\lambda_p < \lambda < \lambda_r$	Slender Compression Element $\lambda > \lambda_r$
Plastic Buckling	Section F1.1	Not Applicable	Not Applicable
Flange Local Buckling (not applicable to tees & double angles)	Not Applicable	Appendix F1	Appendix F1
Web Local Buckling (not applicable to tees & double angles)	Not Applicable	Appendix F1	Appendix F1
Lateral Torsional Buckling (not applicable for members subject to bending about minor axis or to square or circular shapes)	Section F1.2 for doubly-symmetric shapes. Appendix F1. For all other sections		

Limiting Elastic Buckling Moment

$$M_r = F_L S_x \qquad \text{\{Eq. F1-7\}}$$

Where:

F_L = smaller of (F_{yf} - F_r) or F_{yw}, ksi {F1.2a}

F_{yf} = specified minimum flange stress, ksi

F_r = compressive residual stress in flange, 10 ksi for rolled shapes, 16.5 ksi for welded built-up shapes, ksi

F_{yw} = specified minimum web stress, ksi

S_x = elastic section modulus, in^3

FLEXURAL YIELDING LIMIT STATE

- Shapes with compact flanges and webs must meet the requirements for flexural yielding (plastic buckling) limit state.
- The entire beam cross-section can reach the yield stress without buckling of individual elements.

Manual Page 16.1-31. Section F1.1. Yielding:

$$\phi_b = 0.90 \qquad \{F1.1\}$$

$$M_n = M_p \qquad \{Eq.\ F1\text{-}1\}$$

$$M_p = F_y Z \le 1.5 M_y \qquad \{F1.1\}$$

$$M_y = F_y S \qquad \{F1.1\}$$

$$\boxed{F_b = 0.66 F_y} \qquad \boxed{\text{ASD Eq. F1-1}}$$

Where:

M_u = required flexural strength, in-K
ϕ_b = resistance factor, unitless
M_n = nominal flexural strength, in-K
M_p = plastic bending moment, in-K
F_y = specified minimum yield stress, ksi
Z = plastic section modulus, in^3
M_y = elastic bending moment, in-K
S = elastic section modulus, in^3

Note that the consideration of the elastic bending moment is not necessary for sections with shape factors $\left(Z/S\right)$ less than 1.5, such as all W-shape beams bent about their x-axis.

Note that the consideration of the elastic bending moment is necessary for sections with shape factors $\left(Z/S\right)$ more than 1.5, such as all W-shape beams bent about their y-axis.

Manual Page 5-42. Table 5.3. W-Shape Selection By Z_x: AISC design aid is very valuable when yielding limit state is expected to govern.

FLANGE LOCAL BUCKLING LIMIT STATE

- Shapes with noncompact or slender flanges must meet the requirements for flange local buckling.
- The entire beam cross-section can reach the yield stress before local buckling of the compression flange.

Manual Page 16.1-96. Section F1. Design For Flexure:

$$\lambda = \frac{b}{t}$$

$$\lambda_p \; and \; \lambda_r \qquad \text{\{Table B5.1\}}$$

$$M_u \le \phi_b M_n \qquad \text{\{A5.3\}}$$

For $\lambda_p < \lambda \le \lambda_r$:

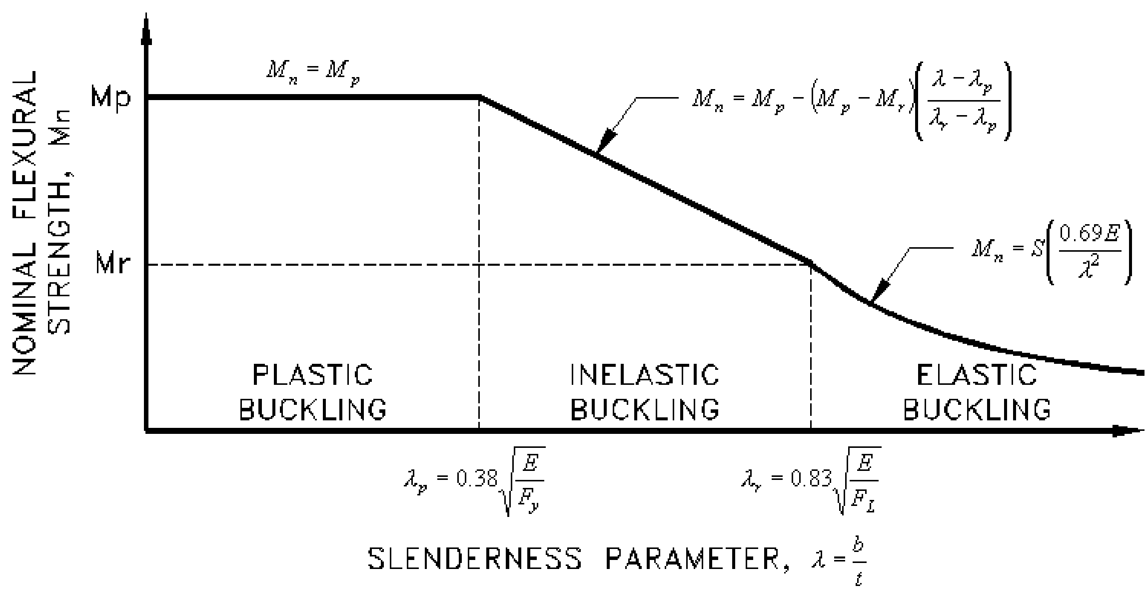
$$M_n = M_p - \left(M_p - M_r\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{\{Eq. A-F1-3\}}$$

$$\boxed{F_b = F_y\left[0.79 - 0.002\frac{b_f}{2t_f}\sqrt{F_y}\right]} \qquad \boxed{\text{ASD Eq. F1-3}}$$

For $\lambda > \lambda_r$:

$$M_n = M_{cr} = SF_{cr} \le M_p \qquad \text{\{Eq. A-F1-4\}}$$

$$M_n = S\left(\frac{0.69E}{\lambda^2}\right) \qquad \text{\{Table A-F1.1\}}$$



FLANGE LOCAL BUCKLING LIMIT STATE – W-SHAPES

WEB LOCAL BUCKLING LIMIT STATE

- Shapes with noncompact or slender webs must meet the requirements for flange local buckling.
- The entire beam cross-section can reach the yield stress before local buckling of the web.

Manual Page 16.1-96. Section F1. Design For Flexure:

$$\lambda = \frac{h}{t_w}$$

$$\lambda_p \text{ and } \lambda_r \qquad \text{\{Table B5.1\}}$$

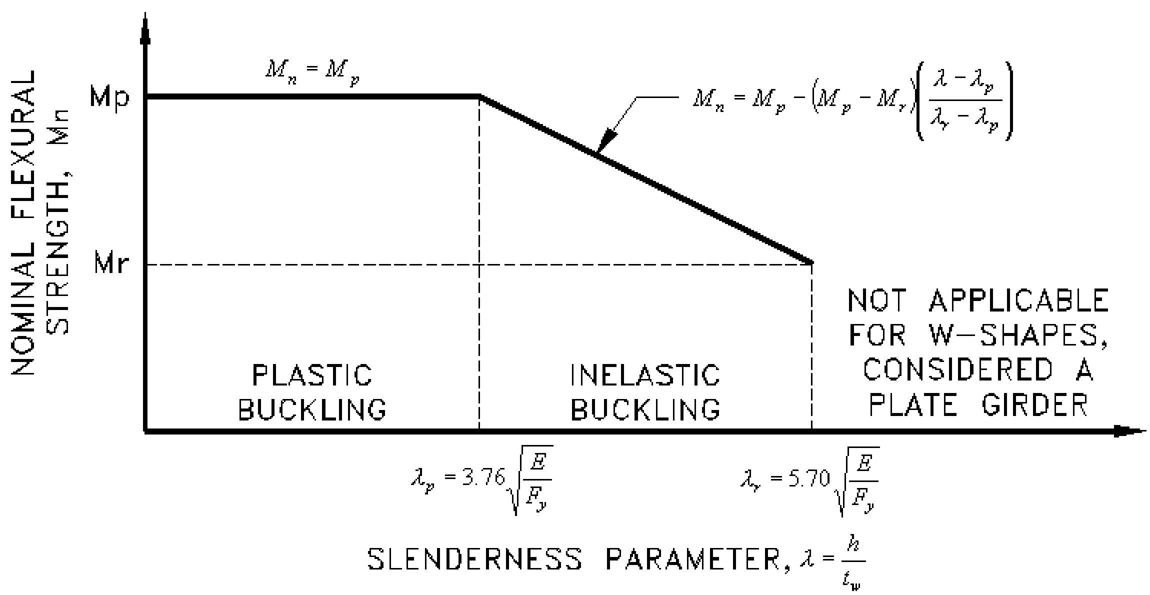
$$M_u \le \phi_b M_n \qquad \text{\{A5.3\}}$$

For $\lambda_p < \lambda \le \lambda_r$:

$$M_n = M_p - \left(M_p - M_r\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{\{Eq. A-F1-3\}}$$

$$\boxed{F_b = 0.60F_y} \qquad \boxed{\text{ASD Eq. F1-5}}$$

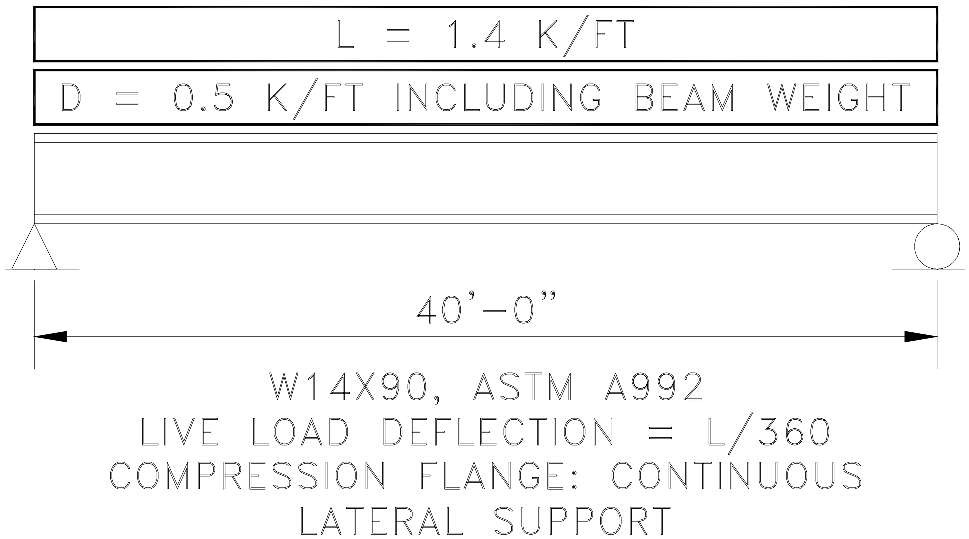
For $\lambda > \lambda_r$: The web local buckling limit state is not applicable. See App. G2.



WEB LOCAL BUCKLING LIMIT STATE – W-SHAPES

EXAMPLE PROBLEM BEAMS #1

GIVEN:



REQUIRED: Check adequacy of beam. Consider flexural yielding and local buckling limit states only. Check deflection.

SOLUTION:

1) Required strength

$$w_u = 1.2(0.5\,K/ft) + 1.6(1.4\,K/ft) = 2.84\,K/ft$$

$$M_u = \frac{2.84K/ft(40\,ft)^2}{8} = 568\,ft - K$$

2) Design strength – flexural yielding

footnote indicates flange is noncompact {Manual 5-46}

<u>flexural yielding is not valid limit state</u>

3) Check flange local buckling

$$\lambda = \frac{b}{2t_f} = 10.2$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000\,ksi}{50\,ksi}} = 9.15$$ {Table B5.1}

$$\lambda_r = 0.83\sqrt{\frac{E}{F_L}} = 0.83\sqrt{\frac{29000\,ksi}{50\,ksi - 10\,ksi}} = 22.35$$ {Table B5.1}

$\lambda_p < \lambda < \lambda_r$ <u>flange is non compact</u>

4) Check web local buckling

$$\lambda = \frac{h}{t_w} = 25.9$$

$$\lambda_p = 3.75\sqrt{\frac{E}{F_y}} = 3.75\sqrt{\frac{29000\,ksi}{50\,ksi}} = 90.31 \qquad \text{\{Table B5.1\}}$$

$\lambda < \lambda_p$ <u>web is compact</u>

5) Design strength – local flange buckling

Calculate $\phi_b M_p$ as if plastic section could be reached.

$$\phi_b M_p = \phi_b F_y Z = \frac{0.90(50\,ksi)(157\,in^3)}{12\,in/ft} = 589\ ft - K$$

$$\phi_b M_r = 429\ ft - K \qquad \text{\{Manual 5-46\}}$$

$$\phi_b M_n = \phi_b M_p - (\phi_b M_p - \phi_b M_r)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) = 589 - (589 - 428)\left(\frac{10.2 - 9.2}{22.3 - 9.2}\right) = 577\ ft - K$$

$M_u = 568\ ft - K < 577\ ft - K = \phi_b M_n$ <u>ok</u>

6) Deflection

$$\Delta_{L(allow)} = \frac{L}{360} = \frac{40\ ft(12\,in/ft)}{360} = 1.33\,in$$

$$\Delta_L = \frac{5(1.4\,K/ft)(40\ ft)^4(1728\,in^3/ft^3)}{384(29000\,ksi)(999\,in^4)} = 2.78\,in > 1.33\,in = \Delta_{L(allow)}$$ <u>ng</u>

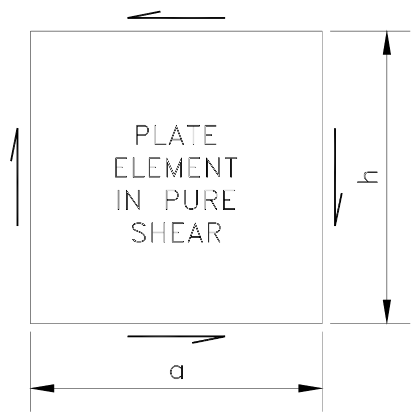
W14x90 Is Not Acceptable

FLEXURAL SHEAR STRENGTH

AISC defines shear strength as a function of web area only.

Web Yielding Limit State

The entire plate cross-section can reach the shear yield stress without shear buckling of the plate.



SHEAR ON SIMPLY SUPPORTED PLATE

Where:

h = vertical plate dimension, in

a = horizontal plate dimension, in

Elastic Web Buckling Limit State

None of the web cross-section can reach the shear yield stress without shear buckling of the entire web. As in all stability situations, slender plates can elastically buckle before yield stresses are reached.

When $\frac{h}{t_w} \le 2.45\sqrt{\frac{E}{F_y}}$: $V_n = 0.6 F_{yw} A_w$ {Eq. F2-1}

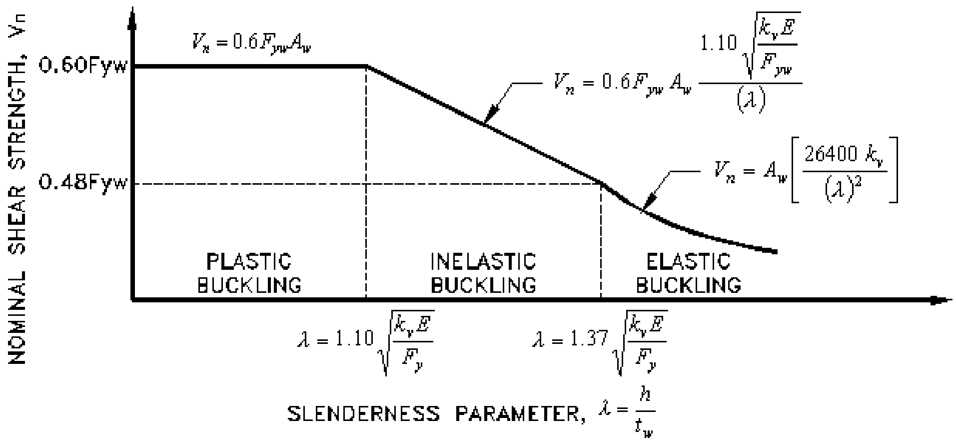
$$\boxed{When \frac{h}{t_w} \le \frac{380}{\sqrt{F_y}} \Rightarrow F_v = 0.40 F_y}$$ ASD Eq. F4-1

Inelastic Buckling Limit State

Some, but not all of the web cross-section can reach the shear yield stress without shear buckling of the entire web. As in all stability situations, residual stresses and imperfections cause inelastic buckling as critical stress approach yield stress.

FLEXURAL SHEAR SUMMARY

Appendix F



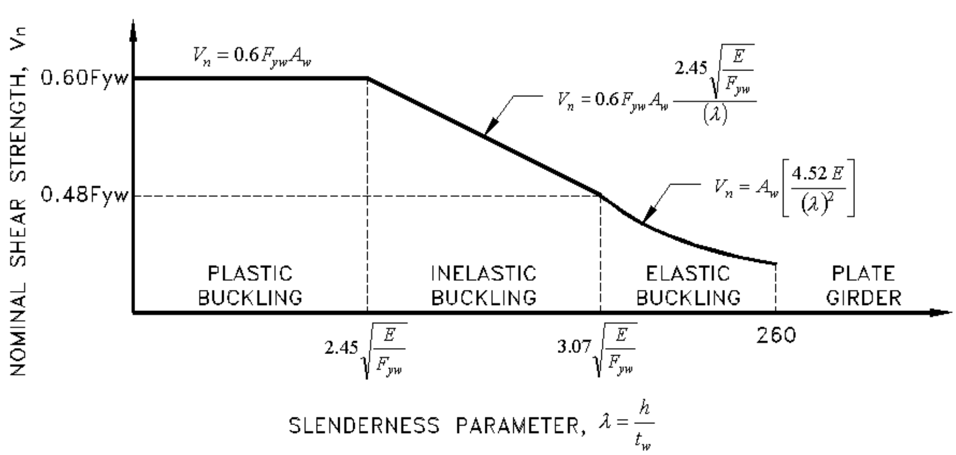
SHEAR BUCKLING LIMIT STATE – APPENDIX F

<u>Manual Page 16.1-152. Table 8-36. Design Shear Stress By Appendix F2 For 36 KSI Steel</u> and <u>Manual Page 16.1-154. Table 8-50. Design Shear Stress By Appendix F2 For 50 KSI Steel</u>: Values of $\dfrac{\phi_v V_n}{A_w}$ are tabulated as a function of a/h and h/t_w ratios. The calculation of k_v is not required if these design tables are used.

$$k_v = 5 + 5\left(a/h\right)^2$$

Chapter F

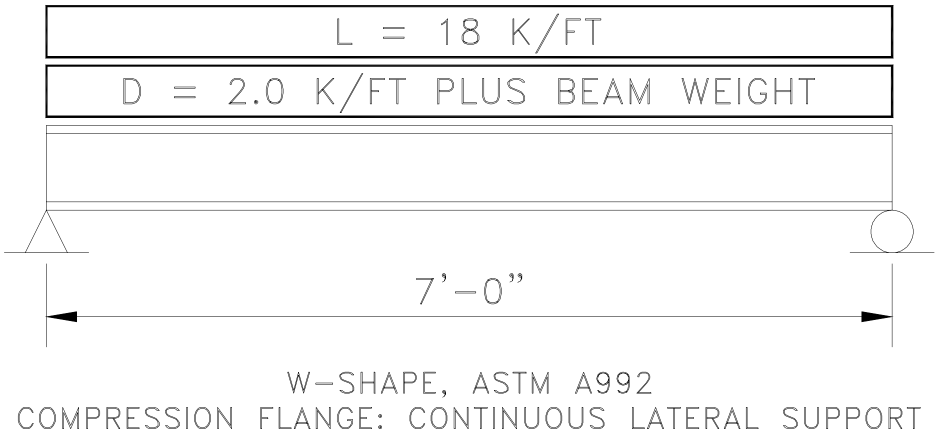
Setting $k_v = 5$ for W-shapes:



SHEAR BUCKLING LIMIT STATE – W-SHAPES

EXAMPLE PROBLEM BEAMS #2

GIVEN:



REQUIRED: Determine lightest section. Consider flexure and shear. Determine live load deflection.

SOLUTION:

1) Required strength

assume beam weighs 100 #/ft

$$w_u = 1.2(2.1K/ft) + 1.6(18K/ft) = 31.3K/ft$$

$$M_u = \frac{31.3K/ft(7\,ft)^2}{8} = 192\,ft-K$$

$$V_u = \frac{31.3K/ft(7\,ft)}{2} = 110K$$

2) Design strength – flexural yielding

Select W16x31 compact {Manual 5-48}

$\phi_b M_n = \phi_b M_p = 203\,ft-K > 192\,ft-K = M_u$ ok {Manual 5-48}

3) Check W16x31 – design strength – shear

$\phi_v V_n = 118K > 110K = V_u$ ok {Manual 5-63)

4) W16x31 – determine live load deflection

$$\Delta_L = \frac{5(18K/ft)(7\,ft)^4\left(1728\,in^3/ft^3\right)}{384(29000\,ksi)\left(375\,in^4\right)} = 0.089\,in$$

Select W16x31, Δ_L = 0.089"

WEAK-AXIS BENDING

Compact Sections – Flexural Yielding

$$\phi_b = 0.90 \qquad \{F1.1\}$$

$$M_n = M_p \qquad \{Eq.\ F1\text{-}1\}$$

$$M_p = F_y Z \le 1.5 M_y \qquad \{F1.1\}$$

$$\boxed{F_b = 0.75 F_y} \qquad \boxed{\text{ASD Eq. F2-1}}$$

$$M_y = F_y S \qquad \{F1.1\}$$

Where:

ϕ_b = resistance factor, unitless
M_n = nominal flexural strength, in-K
M_p = plastic bending moment, in-K
F_y = specified minimum yield stress, ksi
Z = plastic section modulus, in^3
M_y = elastic bending moment, in-K
S = elastic section modulus, in^3

Note that for W-shape beams bent about their y-axis: $\xi_x = shape\ factor = \dfrac{Z_x}{S_x} \ge 1.50$

$F_y Z \le 1.5 F_y S \quad \therefore$ 1.5F_yS will <u>always</u> govern $\phi_b M_p$

Noncompact Or Slender Compression Element Sections - Flange Local Buckling

$$M_n = M_p - \left(M_p - M_r\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \{Eq.\ A\text{-}F1\text{-}2\}$$

Where:

M_r = limiting torsional buckling moment, in-K

$$M_r = F_L S \qquad \{Eq.\ F1\text{-}7\}$$

F_L = smaller of (F_{yf} – F_r) or F_{yw}, ksi {F1.2a}
F_{yf} = specified minimum flange stress, ksi
F_{yw} = specified minimum web stress, ksi

Noncompact Or Slender Compression Element Sections - Web Local Buckling

For beams bent about weak-axis, the web is barely in compression and web local buckling is not a concern.

BIAXIAL BENDING

Interaction Formulae

For an individual structural member, the strength limit state can be summarized as:

$$U = \sum \gamma_i Q_i \leq \phi R_n \qquad \text{\{A5.3\}}$$

Where:

U = required strength, kips or ft-kips
γ_i = load factor, unitless
Q_i = load effect, kips or ft-kips
ϕ = resistance factor, unitless
R_n = nominal resistance, kips or ft-kips
ϕR_n = design strength, kips or ft-kips

This can be written in the following form:

$$\frac{\sum \gamma_i Q_i}{\phi R_n} \text{ or } \frac{\sum Load\ Effects}{\text{Re}\,sis\tan ce} \leq 1.0$$

If more than one type of resistance is involved:

$$\left(\frac{\sum Load\ Effects}{\text{Re}\,sis\tan ce}\right)_1 + \left(\frac{\sum Load\ Effects}{\text{Re}\,sis\tan ce}\right)_2 \leq 1.0$$

<u>If</u> the section has at least on axis of symmetry <u>and</u> is loaded through the centroid, the combined resistances of bending about both the x-axis and y-axis can be expressed as:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1.0$$

Where:

M_{ux} = required x-axis flexural strength, in-K
ϕ_b = flexural resistance factor = 0.90
M_{nx} = nominal x-axis flexural strength, in-K
M_{uy} = required y-axis flexural strength, in-K
M_{ny} = nominal y-axis flexural strength, in-K

Specification Requirements

Manual Page 16.1-38. Section H1. Symmetric Members Subject To Bending And Axial Force: For doubly and singly symmetrical members:

When $\frac{P_u}{\phi P_n} \geq 0.2$: $$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$$ {Eq. H1-1a}

When $\frac{P_u}{\phi P_n} < 0.2$: $$\frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$$ {Eq. H1-1b}

$$\boxed{When \frac{f_a}{F_a} \leq 0.15 \Rightarrow \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0}$$ ASD Eq. H1-3

Where:

- P_u = required axial strength, Kips
- P_n = nominal axial strength, Kips
- ϕ = ϕ_t = 0.90 for gross section yielding
- ϕ = ϕ_t = 0.75 for net section fracture

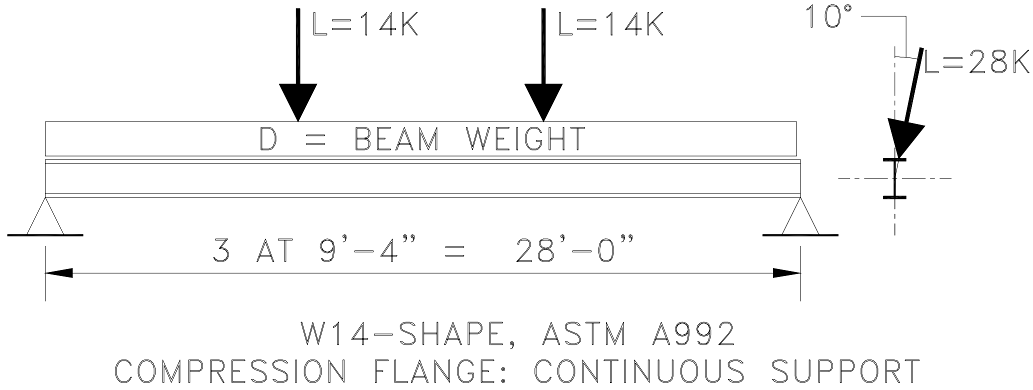
For the case of unsymmetrical bending with no axial loads (P_u = 0 K), Equation H1-1b is applicable, and reduces to:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1.0$$

The Specification clearly permits use of these equations for biaxial bending, even though they were developed for members primarily in axial tension or compression with some bending load.

EXAMPLE PROBLEM BEAMS #3

GIVEN:



REQUIRED: Select beam. Consider shear. Do not consider deflection.

SOLUTION:

1) Required strength

$$P_{ux} = 1.6(14\,K)(\cos 10^\circ) = 22.1K$$

$$P_{uy} = 1.6(14\,K)(\sin 10^\circ) = 3.89K$$

Assume beam weighs 100 #/ft

$$V_{ux} = \frac{1.2(0.10\,K/ft)(28\,ft)}{2} + 22.1K = 23.8\,K$$

$$M_{ux} = \frac{1.2(0.10\,K/ft)(28\,ft)^2}{8} + 22.1K(9.33\,ft) = 218\,ft-K$$

$$M_{uy} = 3.89\,K(9.33\,ft) = 36.3\,ft-K$$

Assume 50% of capacity is available for M_{ux}.

2) Design strength – flexural yielding – x-axis

L_b = 0 ft <u>flexural yielding governs</u>

$$\phi_b M_{n(req)} \approx \frac{218\,ft-K}{0.50} = 436\,ft-K$$

Select W14x68

$$\phi_b M_n = \phi_b M_p = 431\,ft-K \approx 436\,ft-K = \phi_b M_{n(req)}$$ {Manual 5-47}

3) Check W14x68 - design strength - shear

$\phi_v V_n = 157\,K > 23.8\,K = V_u$ ok {Manual 5-66}

4) Check W14x68 - design strength – flexural yielding – y-axis

$$\phi_b M_n = \phi_b F_y Z_y = \frac{0.90(50\,ksi)(36.9\,in^3)}{12\,in/ft} = 138\,ft-K$$ {F1.1}

$$\phi_b M_{n(\max)} = \phi_b 1.5 F_y S_y = 0.90\frac{1.5(50\,ksi)(24.2\,in^3)}{12\,in/ft} = 136\,ft-K$$ governs

5) Check W14x68 interaction

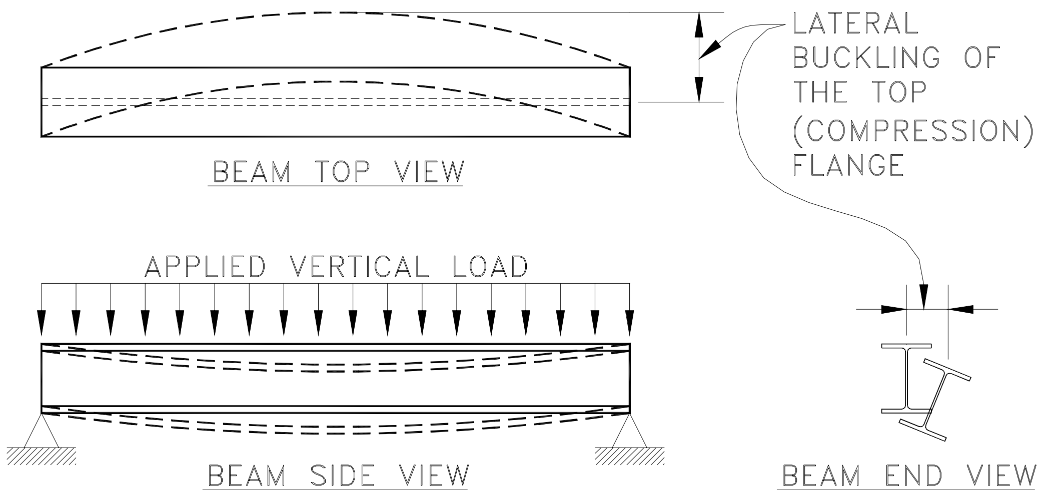
$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{218\,ft-K}{431\,ft-K} + \frac{36.3\,ft-K}{136\,ft-K} = 0.51 + 0.27 = 0.88 < 1.0$$ ok {Eq. H1-1b}

Select W14x68

RATIONAL ANALOGY TO PURE COLUMNS

Beams subject to flexure have much greater strength and stiffness about the strong (x-x) axis than about the weak (y-y) axis.

Unless these members are properly braced against lateral deflection and twisting, they are subject to failure by lateral-torsional buckling about the weak axis prior to reaching their full strong axis capacity.



COMPRESSION FLANGE LATERAL BUCKLING

Consider the compression flange of a laterally unsupported beam. If the compression flange were a pure rectangular column, simply supported for both axes, it would buckle in its weakest direction. However, the compression flange is restrained from buckling in its weakest direction, the beams strong axis, by the continuous support of the beam web.

Therefore, at higher flexural compression loads, the flange would tend to buckle in its strongest direction, the beam's weak axis, twisting the beam. It is this sudden instability in the lateral direction that is referred to as lateral buckling or lateral torsional buckling.

Design procedures must account for the fact that a laterally unsupported beam could experience lateral torsional buckling prior to reaching:

- plastic moment (inelastic buckling).
- the yield moment (elastic buckling)

Some judgement must be used in deciding what can be considered satisfactory lateral support for the compression flange.

LATERAL SUPPORT

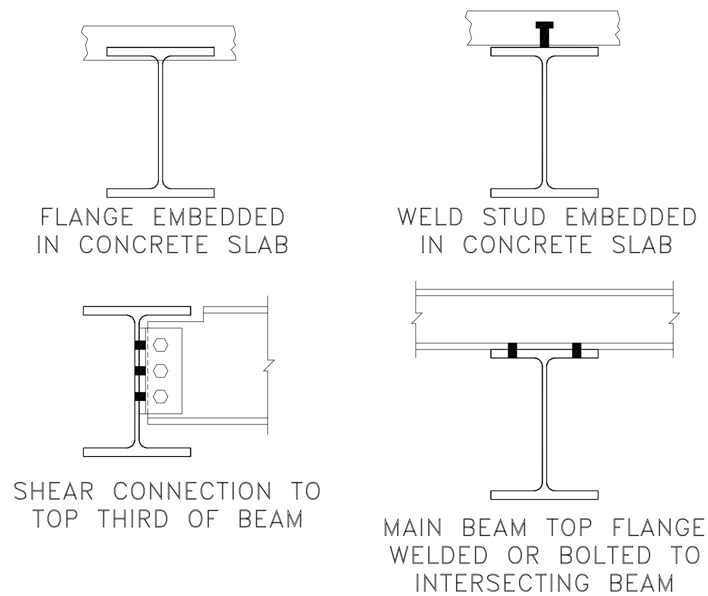
To evaluate the lateral support of a beam, it is necessary to provide some construction examples.

Definite Lateral Support

There are two types of lateral support that are definite and adequate:

- Continuous lateral support by embedment of the compression flange in a concrete floor slab

- Lateral support at intervals provided by cross-beams, cross-frames, ties, or struts. The lateral support system itself must be adequately stiff and braced.



COMPRESSION FLANGE LATERAL SUPPORT

Doubtful Lateral Support

There are several cases that provide doubtful lateral support:

- Bracing that frames nearer the beam tension flange

- Timber decking that rests on the beam, but is not fastened to it, providing only frictional restraint

- Attachment of light gage metal siding or roof decking

It is best in doubtful cases to design the beam as laterally unsupported.

Lateral Bracing Design

Will be discussed later in lecture.

ELASTIC BUCKLING – W-SHAPES

Manual 16.1-31. Section F1. Design For Flexure: Lateral-torsional buckling is not possible if the moment of inertia about the bending axis is equal to or less than the out-of-plane moment of inertia and is is not possible for:

- Shapes bent about their minor axis
- Shapes with $I_x < I_y$
- Circular or square shapes

The elastic lateral-torsional buckling strength, M_{cr}, for an I-shaped beam section under the action of a uniform moment in the plane of the web over the laterally unbraced length, L is as follows:

$$M_{cr} = \frac{\pi}{L}\sqrt{\left(\frac{\pi E}{L}\right)^2 C_w I_y + EI_y GJ}$$

The first term represents the warping torsion and the second term represents the pure torsion of the section.

Where:

C_w = warping constant, in^6
I_y = weak-axis moment of inertia, in^4
G = shear modulus = 11200 ksi
J = torsional constant, in^4

Compression Flange Limits

$$L_b > L_r$$

Where:

L_r = limiting laterally unbraced length for inelastic lateral torsional buckling, inch

$$L_r = \frac{r_y X_1}{F_L}\sqrt{1+\sqrt{1+X_2 F_L{}^2}} \quad \text{\{Eq. F1-6\}}$$

r_y = radius of gyration about y-axis, inch
X_1 = beam buckling factor, ksi

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} \quad \text{\{Eq. F1-8\}}$$

X_2 = beam buckling factor, in^4/K^2

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right) \quad \text{\{Eq. F1-9\}}$$

F_L = smaller of ($F_{yf} – F_r$) or F_{yw}, ksi {F1.2a}

F_r = specified residual stress, ksi

S_x = elastic section modulus about x-axis, in^3

A = cross-sectional area, in^2

Flexural Strength

$$M_r > M_n$$

Where:

M_n = nominal flexural strength, in-K

$$M_n = M_{cr} \le M_p \qquad \text{\{Eq. F1-12\}}$$

M_r = limiting torsional buckling moment, in-K

$$M_r = F_L S_x \qquad \text{\{Eq. F1-7\}}$$

M_p = plastic bending moment, in-K

M_{cr} = critical elastic moment, in-K

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \qquad \text{\{Eq. F1-13\}}$$

$$\boxed{F_b = \left[\frac{2}{3} - \frac{F_y\left(L/r_T\right)^2}{1530000C_b}\right]F_y \le 0.60F_y} \qquad \boxed{\text{ASD Eq. F1-6}}$$

$$\boxed{F_b = \frac{170000C_b}{\left(L/r_T\right)^2} \le 0.60F_y} \qquad \boxed{\text{ASD Eq. F1-7}}$$

$$\boxed{F_b = \frac{12000C_b}{L\left(d/A_f\right)} \le 0.60F_y} \qquad \boxed{\text{ASD Eq. F1-8}}$$

$$M_{cr} = \frac{C_b S_x X_1 \sqrt{2}}{\left(\frac{L_b}{r_y}\right)}\sqrt{1 + \frac{X_1^{\,2} X_2}{2\left(\frac{L_b}{r_y}\right)^2}} \qquad \text{\{F1.2b\}}$$

Letting $\lambda = \frac{L_b}{r_y}$: $\quad M_{cr} = \frac{C_b S_x X_1 \sqrt{2}}{(\lambda)}\sqrt{1 + \frac{X_1^{\,2} X_2}{2(\lambda)^2}}$

C_b = bending coefficient, unitless

λ = slenderness coefficient, unitless

BENDING COEFFICIENTS

Lateral-torsional buckling design moment strength equations are based on the critical buckling strength of the compression flange.

These equations assume that the compression flange is at the same stress level for its entire length. However, this is only true for a constant moment. Therefore, the design strength must be adjusted upwards for other moment gradients.

AISC has developed an expression for C_b to adjust the beam moment gradients:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad \{\text{Eq. F1-3}\}$$

$$\boxed{C_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3} \quad \boxed{\text{ASD Sec. F1.3}}$$

Where:

M_{max} = absolute value of maximum moment in the unbraced segment
M_A = absolute value of moment at quarter point of unbraced segment
M_B = absolute value of moment at centerline of unbraced segment
M_C = absolute value of moment at three-quarter point of unbraced segment

An unbraced segment is defined as "the beam segment bounded by adjacent lateral support points of the beam compression flange".

- A single piece of steel used as a beam can be considered to be multiple beam segments for analysis purpose, depending on the location of compression flange lateral supports.
- This is similar to a piece of steel used as a column which may stretch for two stories, but is treated as two column segments for analysis purposes.

Note that for a constant moment diagram, $C_b = 1.0$

Manual Page 16.1-32. Doubly Symmetric Shapes and Channels with $L_b \le L_r$:

- AISC permits the use of $C_b = 1.0$ for all cases, because it is conservative.
- AISC requires that $C_b = 1.0$ for cantilevers or overhangs where the free end compression flange is unbraced.

Manual Page 5-35. Table 5-1. Values Of C_b For Simply Supported Beams: For convenience, AISC tabulates certain common values.

INELASTIC BUCKLING – W-SHAPES

Some, but not all, of the beam cross-section can reach the yield stress without buckling of individual elements or lateral-torsional buckling of the entire beam.

Compression Flange Limits

$$L_p < L_b < L_r$$

Where:

L_p = limiting laterally unbraced length for full moment capacity uniform moment case $C_b = 1.0$ inch

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_{yf}}} \qquad \text{\{Eq. F1-4\}}$$

$$\boxed{L_u = bigger\ of\ \frac{20000}{\left(\frac{d}{A_f}\right)F_y}\ or\ r_T\sqrt{\frac{102000}{F_y}}} \qquad \boxed{\text{ASD Sec. F1.3}}$$

Flexural Strength

$$M_p \geq M_n > M_r$$

$$M_n = C_b\left[M_p - \left(M_p - M_r\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \leq M_p$$

$$M_n = C_b\left[M_p - \left(M_p - M_r\right)\left(\frac{L_b - L_r}{L_r - L_p}\right)\right] \leq M_p \qquad \text{\{Eq. F1-2\}}$$

$$\phi_b M_n = C_b\left\lfloor \phi_b M_p - BF\left(L_b - L_p\right)\right\rfloor \leq \phi_b M_p$$

$$BF = \frac{\phi_b M_p - \phi_b M_r}{L_r - L_p}$$

Where:

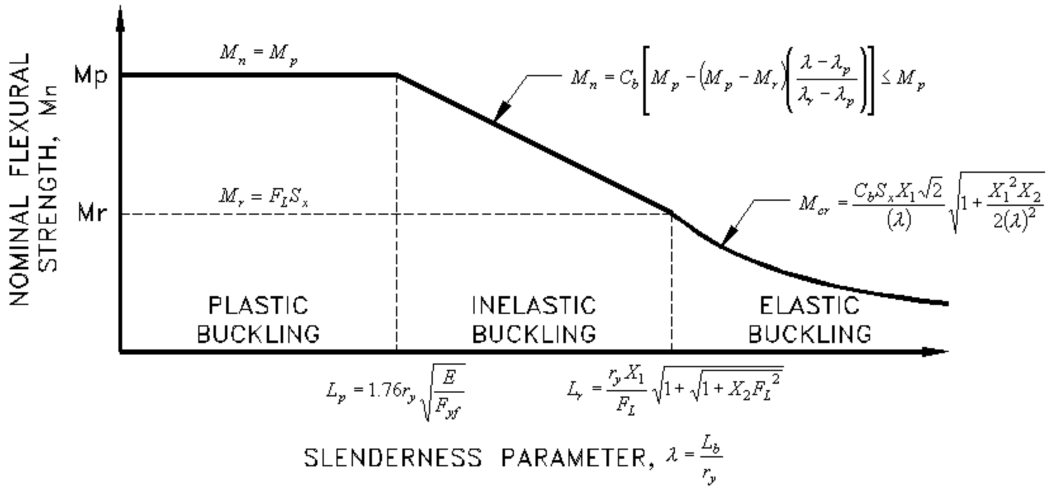
λ_p = slenderness coefficient, unitless

$$\lambda_p = \frac{L_p}{r_y}$$

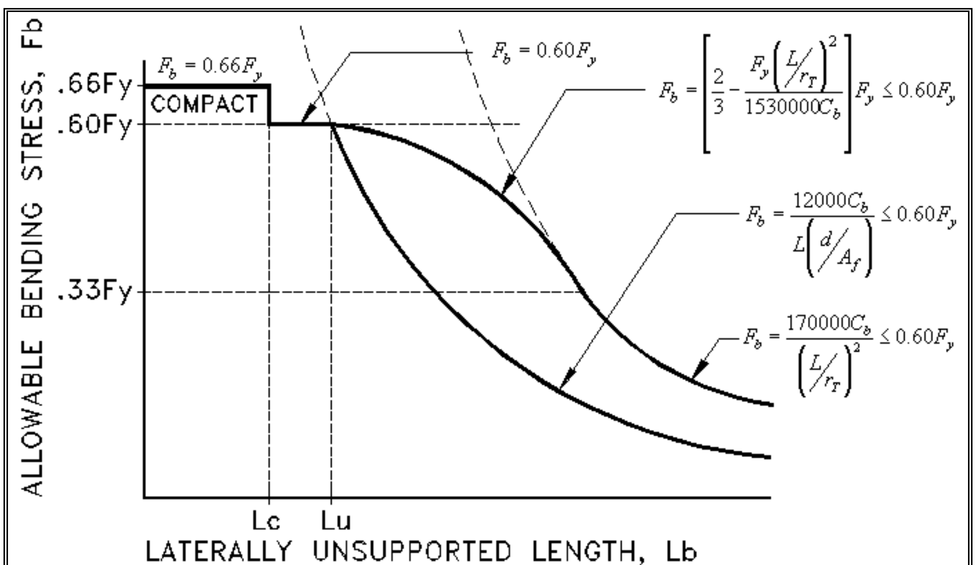
λ_r = slenderness coefficients, unitless

$$\lambda_r = \frac{L_r}{r_y}$$

SUMMARY – LTB FOR W-SHAPES



LATERAL TORSIONAL BUCKLING LIMIT STATE – W-SHAPES
SHOWN WITH Cb = 1.0



STRONG AXIS BENDING, I-SHAPED MEMBERS

Trial Beam Selection

If the C_b value is very high or $L_b \approx L_r$, it is recommended that a first trial for design be selected based on $M_u \le M_n = \phi_b M_p$. If the C_b value is very high or $L_b >> L_r$, it is recommended that a first trial for design be selected based on $M_u \le M_n = \phi_b M_r$.

Manual Page 5-42. Table 5-3. W-shapes Selection By Z_X: Note that values for $\phi_b M_p$, $\phi_b M_r$, L_p, L_r, *BF*, and $\phi_v V_n$ are tabulated.

EXAMPLE PROBLEM BEAMS #4

GIVEN: W12x26, ASTM A992

REQUIRED:

1) Determine L_p, X_1, X_2, and L_r. Compare with tabulated values.
2) Plot $M_u = \phi_b M_n$ vs L_b relationship for C_b = 1.0 and 2.3 and compare with AISC design aids.

SOLUTION:

1a) Determine L_p

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_{yf}}} = 1.76(1.51\,in)\sqrt{\frac{29000\,ksi}{50\,ksi}} = 64.00\,in = 5.33\,ft \qquad \text{\{Eq. F1-4\}}$$

$$L_p = 5.33\,ft \qquad \text{\{Manual 5-48\}}$$

1b) Determine X_1

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} \qquad \text{\{Eq. F1-8\}}$$

$$= \frac{\pi}{33.4\,in^3}\sqrt{\frac{(29000\,ksi)(11200\,ksi)(0.30\,in^4)(7.65\,in^2)}{2}} = 1816\,ksi$$

$$X_1 = 1820\,ksi \qquad \text{\{Manual 1-21\}}$$

1c) Determine X_2

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 \qquad \text{\{Eq. F1-9\}}$$

$$= 4\frac{607\,in^6}{17.3\,in^4}\left(\frac{33.4\,in^3}{(11200\,ksi)(0.30\,in^4)}\right)^2 = 0.0139\frac{in^4}{K^2}$$

$$X_2 = 13900x10^{-6}\,(1/ksi)^2 \qquad \text{\{Manual 1-21\}}$$

1d) Determine L_r

$$F_L = 50\,ksi - 10\,ksi = 40\,ksi$$

$$L_r = \frac{r_y X_1}{F_L}\sqrt{1+\sqrt{1+X_2 F_L{}^2}} \qquad \text{\{Eq. F1-6\}}$$

$$= \frac{1.51in(1816ksi)}{40\,ksi}\sqrt{1+\sqrt{1+\left(0.0139\frac{in^4}{K^2}\right)(40\,ksi)^2}} = 165.4\,in = 13.78\,ft$$

$$L_r = 13.8\,ft \qquad \text{\{Manual 5-48\}}$$

L_p = 5.34 ft, X_1 = 1816 ksi, X_2 = 13900 x 10^{-6} $(1/ksi)^2$, L_r = 13.78 ft

2a) Flexural yielding limit state

$L_b < L_p$

$$\phi_b M_n = \phi_b F_y Z_x = \frac{0.9(50\,ksi)(37.2\,in^3)}{12\,in/ft} = 139.5\,ft-K$$

$$\phi_b M_n = \phi_b M_p = 140\,ft-K$$ {Manual 5-48}

2b) Inelastic buckling limit state

$5.3\,ft = L_p < L_b < L_r = 13.8\,ft$

$$\phi_b M_r = \phi_b F_L S_x = \frac{0.9(40\,ksi)(33.4\,in^3)}{12\,in/ft} = 100.2\,ft-K$$

$$\phi_b M_r = 100\,ft-K$$ {Manual 5-48}

$$\phi_b M_n = C_b\left[\phi_b M_p - (\phi_b M_p - \phi_b M_r)\left(\frac{L_b - L_r}{L_r - L_p}\right)\right] \le \phi_b M_p$$ {Eq. F1-2}

$$= C_b\left[140 - (140 - 100)\left(\frac{L_b - 5.3}{13.8 - 5.3}\right)\right] = C_b(164.9 - 4.71L_b) \le 140\,ft-K$$

$$= 1.0(164.9 - 4.71L_b) = 164.9 - 4.71L_b \le 140\,ft-K$$ {for C_b = 1.0}

$$= 2.3(164.9 - 4.71L_b) = 379.4 - 10.83L_b \le 140\,ft-K$$ {for C_b = 2.3}

Where:

$\phi_b M_n$, ft-K
L_b, ft

2c) Elastic buckling limit state

$13.8\,ft = L_r < L_b$

$$\phi_b M_n = \phi_b M_{cr} = \phi_b \frac{C_b S_x X_1 \sqrt{2}}{\left(\frac{L_b}{r_y}\right)}\sqrt{1 + \frac{X_1^2 X_2}{2\left(\frac{L_b}{r_y}\right)^2}} \le \phi_b M_p$$ {Eq. F1-13}

$$= 0.90\frac{C_b(33.4\,in^3)(1820\,ksi)\sqrt{2}}{\left(\frac{L_b}{1.51\,in}\right)}\sqrt{1 + \frac{(1820\,ksi)^2\left(0.0139\frac{in^4}{K^2}\right)}{2\left(\frac{L_b}{1.51\,in}\right)^2}} \le 140\,ft-K$$

$$= \frac{116829C_b}{L_b}\sqrt{1+\frac{52491}{(L_b)^2}} \le 140\ ft-K$$

$$= \frac{116829(1.0)}{L_b}\sqrt{1+\frac{52491}{(L_b)^2}} = \frac{116829}{L_b}\sqrt{1+\frac{52491}{(L_b)^2}} \le 140\ ft-K \qquad \{\text{for } C_b = 1.0\}$$

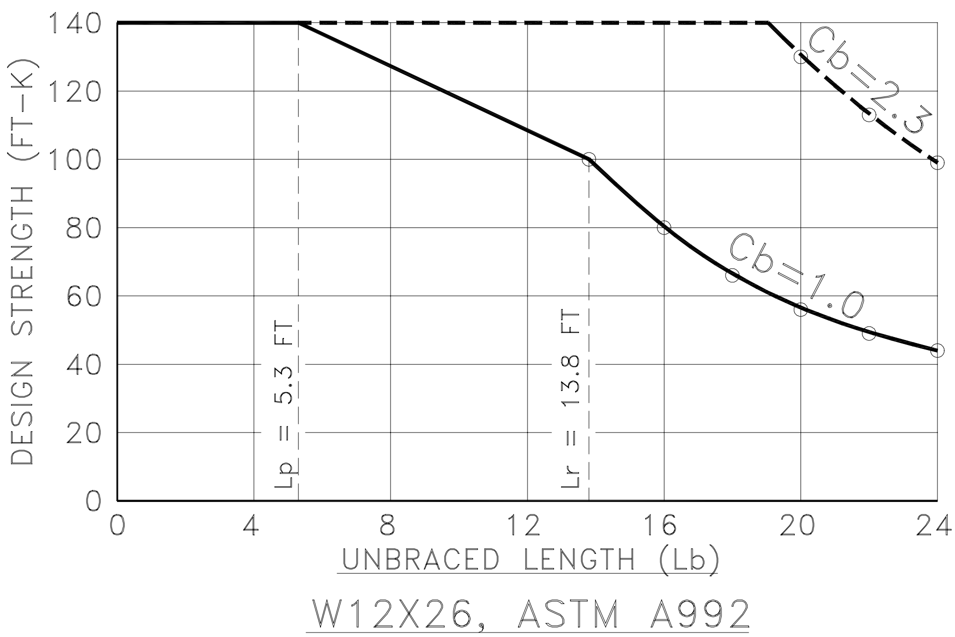
$$= \frac{116829(2.3)}{L_b}\sqrt{1+\frac{52491}{(L_b)^2}} = \frac{268707}{L_b}\sqrt{1+\frac{52491}{(L_b)^2}} \le 140\ ft-K \qquad \{\text{for } C_b = 2.3\}$$

Where:

$\phi_b M_n$, in-K

L_b, in

2e) Equation plot

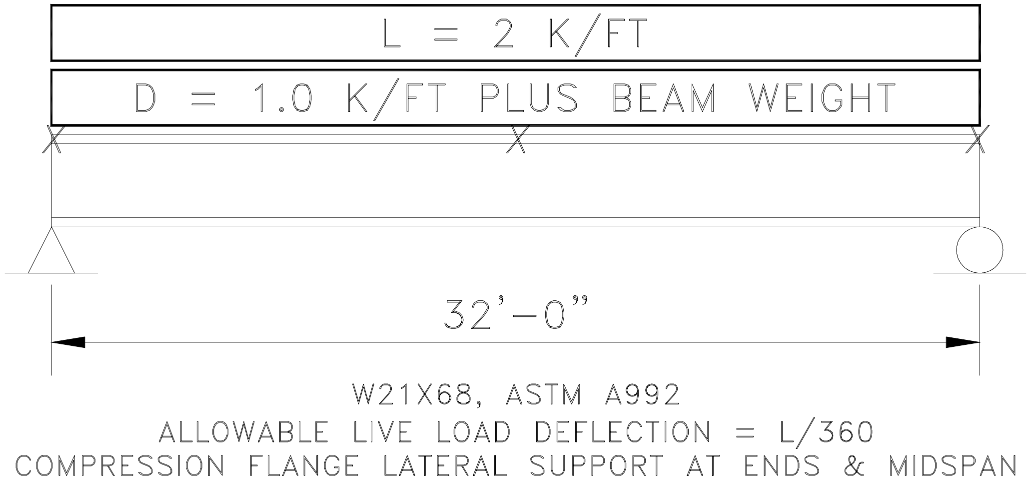


2f) AISC design aid

{Manual 5-100}		
W12x26, ASTM A992, C_b = 1.00		
Data Point		Remarks
$\phi_b M_n$ (ft-K)	L_b (ft)	
140	0	Continuous support
140	5.3	L_p = 5.3 ft, indicated by ●
100	13.8	L_r = 13.8 ft, indicated by ❍
78	16	
63	18	Chart stops

EXAMPLE PROBLEM BEAMS #5

GIVEN:



REQUIRED: Check adequacy of beam. Consider flexure, shear, and deflection.

SOLUTION:

1) Required strength

$$w_u = 1.4(1.068\,K/ft) = 1.50\,K/ft$$

$$w_u = 1.2(1.068\,K/ft) + 1.6(2\,K/ft) = 4.48\,K/ft$$ <u>governs</u>

$$M_u = \frac{4.48\,K/in(32\,ft)^2}{8} = 573\,ft-K$$

$$V_u = \frac{4.48\,K/in(32\,ft)}{2} = 71.7\,K$$

2) Design strength - flexural yielding

$L_p = 6.4\,ft$ {Manual 5-46}

$L_r = 17.3\,ft$ {Manual 5-46}

$L_p < L_b = 16\,ft < L_r$ <u>inelastic buckling</u>

$\phi_b M_n = \phi_b M_p = 600\,ft-K$ {Manual 5-46}

3) Design strength - lateral torsional buckling

$C_b = 1.30$ for each segment {Manual 5-35}

$\phi_b M_n = C_b(\phi_b M_n)_{chart} = 1.30(441\, ft - K) = 573\, ft - K$ {Manual 5-92}

4) Design strength – flexure

lateral torsional buckling limit state governs flexure

$M_u = 573\, ft - K \leq 573\, ft - K = \phi_b M_n$ ok

5) Check W21x68 - design strength - shear

$\phi_v V_n = 245\, K > 71.7\, K = V_u$ ok {Manual 5-60}

6) Check W21x68 - deflection

$$\Delta_L = \frac{5wL^4}{384EI} = \frac{5(2.0\, K/ft)(32\, ft)^4(1728\, in^3/ft^3)}{384(29000\, ksi)(1480\, in^4)} = 1.10\, in$$

$$\Delta_{L(allow)} = \frac{L}{360} = \frac{32\, ft(12\, in/ft)}{360} = 1.07\, in < 1.10\, in = \Delta_L \quad \text{ng}$$

W21x68 Is Unacceptable For Deflection

TYPES OF BRACING SYSTEMS

Nodal Column Bracing

The current LRFD specification is the first AISC specification that includes column brace strength and stiffness requirements.

$$P_{br} = 0.01P_u \quad \text{\{Eq. C3-5\}}$$

$$\beta_{br} = \frac{8P_u}{\phi L_b} \quad \text{\{Eq. C3-6\}}$$

Where:

P_{br} = required bracing strength, Kips
P_u = required column strength, Kips
β_{br} = required bracing stiffness, K/in
L_b = brace spacing = unbraced length of compression flange, in
ϕ = resistance factor = 0.75

Nodal Beam Bracing

The current LRFD specification is the first AISC specification that includes beam compression flange brace strength and stiffness requirements.

- For single curvature, bracing must be attached near the compression flange. (Exception: Bracing of cantilevers must be attached near the tension flange.)
- For reverse curvature loading, the required bracing must be attached near both flanges.
- Inflection points may not be considered a brace point.

$$P_{br} = \frac{0.02M_uC_d}{h_o} \quad \text{\{Eq. C3-9\}}$$

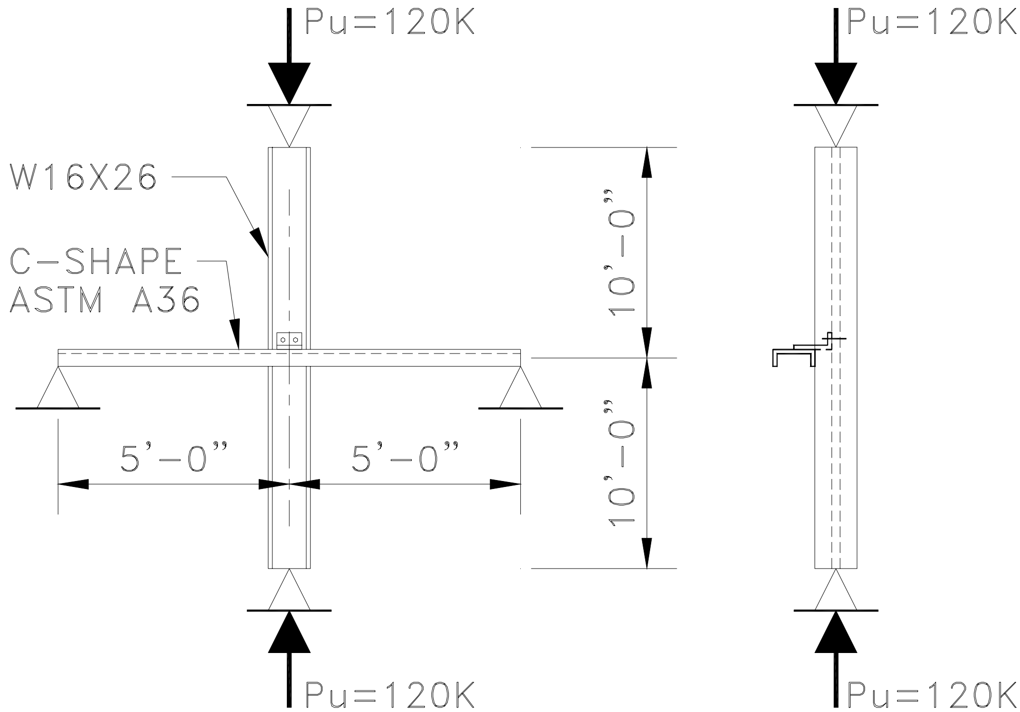
$$\beta_{br} = \frac{10M_uC_d}{\phi L_b h_o} \quad \text{\{Eq. C3-10\}}$$

Where:

M_u = required beam flexural strength, Kips
C_d = curvature coefficient, unitless
= 1.0 for single curvature bending
= 2.0 for reverse curvature bending
h_o = distance between flange centroids, inch

EXAMPLE PROBLEM BEAMS #6

GIVEN:



REQUIRED: Select C-shape to provide weak-axis bracing for column.

SOLUTION:

1) Required strength – nodal column bracing

$$P_{br} = 0.01P_u = 0.01(120\,K) = 1.20K \qquad \text{\{Eq. C3-5\}}$$

2) Required stiffness – nodal column bracing

$$\beta_{br} = \frac{8P_u}{\phi L_b} = \frac{8(120\,K)}{0.75(120\,in)} = 10.7\,K/in \qquad \text{\{Eq. C3-6\}}$$

3) Design stiffness

for single span beam with point load at center

$$\Delta = \frac{PL^3}{48EI}$$ {Manual 5-164}

$$\beta_{br} = \frac{P_{br}}{\Delta} = \frac{48EI}{L^3}$$

$$I_{x(req)} = \frac{B_{br}L^3}{48E} = \frac{10.7\,K/in(120\,in)^3}{48(29000\,ksi)} = 13.3\,in^4$$

<u>Try C6x10.5</u>

$$I_x = 15.1\,in^4 > 13.3\,in^4 = I_{x(req)}$$

4) Design strength – flexural yielding – x-axis - C6x10.5

$$M_u = \frac{P_{br}L}{4} = \frac{1.2K(10\,ft)}{4} = 3.0\,ft-K$$

$$L_p = 2.20\,ft$$ {Manual 5-122}

$$L_r = 12.3\,ft$$ {Manual 5-122}

$L_p < L_b = 5\,ft < L_r$ <u>inelastic buckling</u>

$$\phi_b M_n = \phi_b M_p = 16.7\,ft-K$$ {Manual 5-122}

5) Design strength – lateral torsional buckling – x-axis - C6x10.5

$$C_b = 1.67$$ {Manual 5-35}

AISC beam charts do not cover channels this small {Manual 5-131}

$$M_n = C_b\left[M_p - \left(M_p - M_r\right)\left(\frac{L_b - L_r}{L_r - L_p}\right)\right] \le M_p$$ {Eq. F1-2}

$$\phi_b M_n = C_b\left[\phi_b M_p - BF\left(L_b - L_p\right)\right]$$

$$= 1.67[16.7\,ft-K - 0.680(5.0\,ft - 2.20\,ft)] = 24.7\,ft-K$$

6) Design strength – flexure – x-axis – C6x10.5

flexure is governed by flexural yielding limit state

$\phi M_n = 16.7\,ft-K > 3.0\,ft-K = M_u$ <u>ok</u>

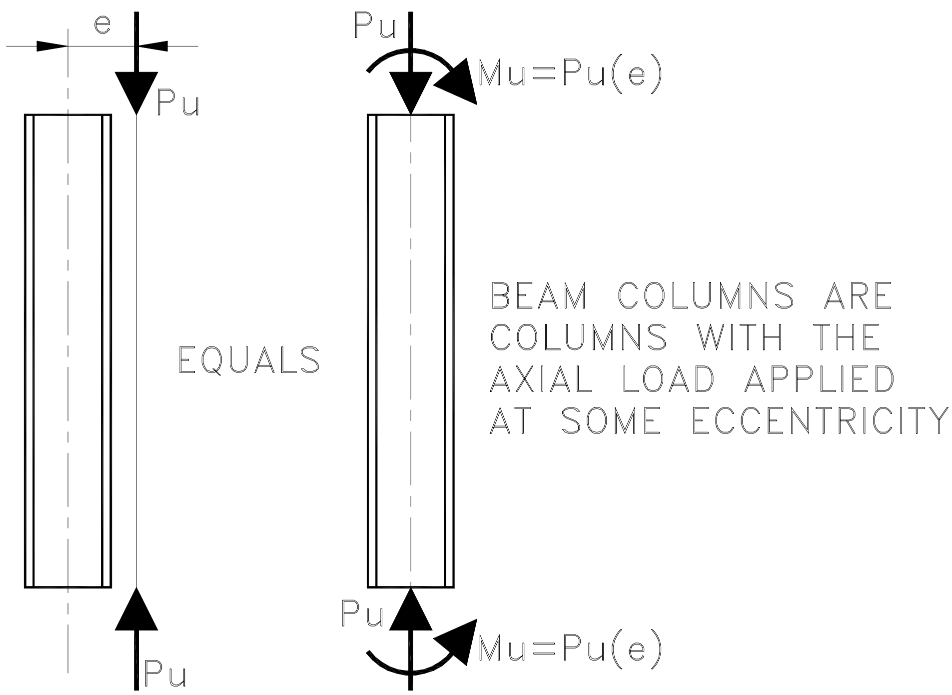
Select C6x10.5 Brace

BEAM-COLUMNS

GENERAL

Beam-columns commonly occur in many steel structures:

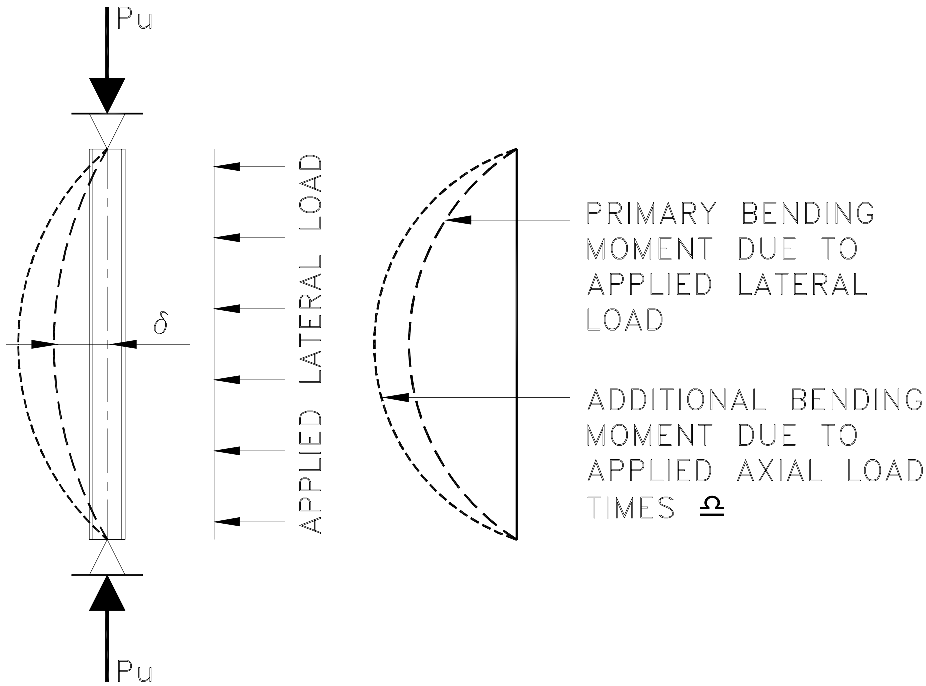
- Loads applied to column with eccentricity due to construction tolerances or connection details
- Rigid frame action of lateral wind and seismic loads
- Rigid frame action of vertical dead and live loads when the loads or the frame are not symmetric
- Top chords of roof trusses supporting roof vertical loads between panel points in addition to the axial loads from truss action
- Bottom chords of roof trusses supporting interior ceilings and light fixtures between panel points in addition to the axial loads from truss action



If the eccentricity (e) is large, the member will behave like a beam. If the eccentricity is small, the member will behave like a column.

Moments concurrent with axial tension loads are not of significant concern because the axial tension tends to reduce compressive buckling tendencies.

MAGNIFICATION FACTOR - BRACED FRAMES



BRACED FRAME SECOND ORDER EFFECTS

An applied lateral load causes a deflection δ in the column and a corresponding bending moment, M_u. The applied axial load times the deflection δ causes additional bending moment, M_u. This additional bending moment is called a second order effect and must be accounted for in beam column design.

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} \geq 1 \qquad \text{\{Eq. C1-2\}}$$

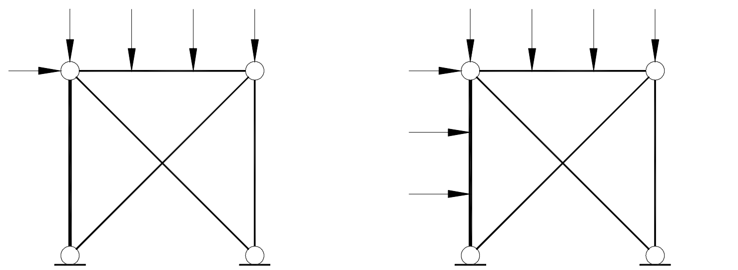
$$P_{e1} = \frac{\pi^2 EI}{(KL)^2} \qquad \text{\{C1.2\}}$$

Where:

- B_1 = magnification factor, unitless
- C_m = modification factor, a function of moment curvature, unitless
- P_u = required axial strength, Kips
- P_{e1} = elastic Euler buckling load, Kips
- E = steel modulus of elasticity = 29,000 ksi
- I = member moment of inertia in plane of bending moment, in^4
- K = effective length factor in plane of bending moment, unitless
- L = member unbraced length in plane of the bending moment, inch

MODIFICATION FACTOR – BRACED FRAME

There are two categories of braced frames that must be considered, those with no transverse loads between supports, and those with transverse loads between supports.



With No Transverse Loads Between Supports

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \qquad \text{\{Eq. C1-3\}}$$

Where:

C_m = moment modification factor, a coefficient based on elastic first-order analysis assuming no lateral translation of the frame, unitless

$\frac{M_1}{M_2}$ = the ratio of the smaller end moment to the larger end moment in the plane of bending under consideration, positive when the member is bent in reverse curvature, negative when bent in single curvature, unitless

With Transverse Loads Between Supports

For members whose ends are restrained:

C_m = 0.85 {C1(b)}

For members whose ends are not restrained:

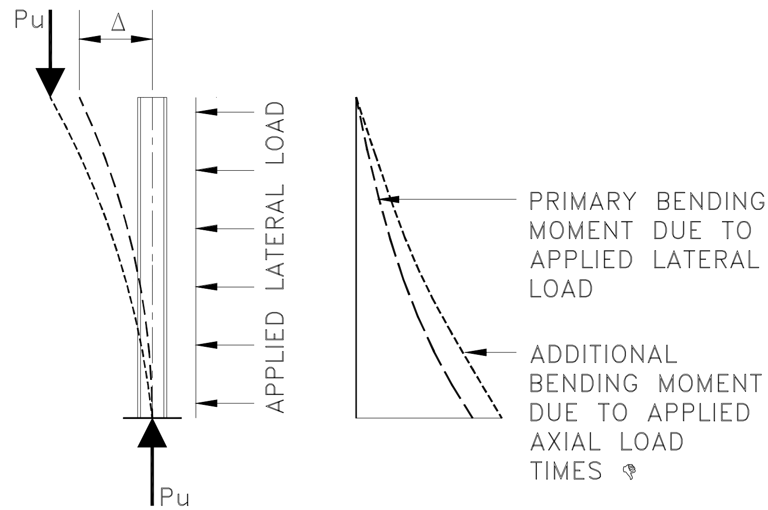
C_m = 1.00 {C1(b)}

Manual Page 16.1-187: For beam columns with transverse loads between supports, the moment modification factor can be approximated as:

$$C_m = 1 + \frac{\Psi P_u}{P_{e1}}$$

Manual Page 16.1-187. Table C-C1.1. Amplification Factors Ψ and C_m

MAGNIFICATION FACTOR - UNBRACED FRAMES



UNBRACED FRAME – SECOND ORDER EFFECTS

An applied lateral load causes a deflection Δ in the column and a corresponding bending moment, M_u. The applied axial load times the deflection Δ causes additional bending moment, M_u. This additional bending moment is called a second order effect and must be accounted for in beam column design.

$$B_2 = \frac{1}{1 - \sum P_u \left(\frac{\Delta_{oh}}{\sum HL} \right)} \qquad \text{\{Eq. C1-4\}}$$

or

$$B_2 = \frac{1}{1 - \frac{\sum P_u}{\sum P_{e2}}} \qquad \text{\{Eq. C1-5\}}$$

$$P_{e2} = \frac{\pi^2 EI}{(KL)^2} \qquad \text{\{C1.2\}}$$

Where:

ΣP_u = required axial strength of all columns in a story, Kips

Δ_{oh} = lateral inter-story deflection, inch

ΣH = sum of all story horizontal forces producing Δ_{oh}, Kips

P_{e2} = elastic Euler buckling load, Kips

Equation C1-4 uses the code maximum horizontal displacement (drift), and all columns can be considered individually.

$$\text{Drift Ratio} = \frac{\Delta_{oh}}{L}$$

Equation C1-5 assumes that all story columns will sidesway simultaneously.

SPECIFICATION REQUIREMENTS

For doubly and singly symmetrical members:

When $\dfrac{P_u}{\phi P_n} \geq 0.2$:
$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$$ {Eq. H1-1a}

$$\boxed{When \frac{f_a}{F_a} > 0.15 \Rightarrow \frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0}$$ ASD Eq. H1-1

and
$$\boxed{\frac{f_a}{0.60 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0}$$ ASD Eq. H1-2

When $\dfrac{P_u}{\phi P_n} < 0.2$:
$$\frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$$ {Eq. H1-1b}

$$\boxed{When \frac{f_a}{F_a} \leq 0.15 \Rightarrow \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0}$$ ASD Eq. H1-3

Where:

P_n = nominal axial strength, Kips

$\phi = \phi_t = 0.90$ for gross section yielding

$\phi = \phi_t = 0.75$ for net section fracture

$\phi = \phi_c = 0.85$ for compression

$\phi_b = 0.90$ for flexure

M_{ux} = required x-axis flexural strength, including second order effects, in-K

M_{uy} = required y-axis flexural strength, including second order effects, in-K

M_{nx} = nominal x-axis flexural strength, in-K

M_{ny} = nominal y-axis flexural strength, in-K

Second Order Effects

$$M_u = B_1 M_{nt} + B_2 M_{lt}$$ {Eq. C1-1}

Where:

M_u = required flexural strength, in-K

M_{nt} = required flexural strength for braced frame (nt = no translation), in-K

B_2 = magnification factor, unitless

M_{lt} = required flexural strength for unbraced frame (lt = lateral translation), in-K

TRIAL SELECTION METHODS

The nature of beam-column analysis is such that direct mathematical solutions are complex. The design of a beam-column is accomplished by analysis of a trial section.

ASD Trial Selection Method

$$P_{eff} = P_o + M_x m + M_y mU$$

Where:

P_0 = actual axial load, Kips
M_x = x-axis bending moment, ft-K
M_y = y-axis bending moment, ft-K
M = factor from Table B, unitless
U = factor from column load tables, unitless

Yura Trial Selection Method

The Yura trial selection methods is based on beam-columns with a dominant axial load.

$$P_{u(equiv)} = P_u + M_{ux}\left(\frac{2}{d}\right) + M_{uy}\left(\frac{7.5}{b_f}\right)$$

Where:

$P_{u(equiv)}$ = equivalent required axial strength, Kips
M_{ux} = required x-axis flexural strength, in-Kips
M_{uy} = required y-axis flexural strength, in-Kips
d = member depth, inch
b_f = member flange width, inch

Alternate Trial Selection Method

The Alternate trial selection methods is based on beam-columns with a dominant flexural load. Beam-columns with dominant flexural loads will be more economical using sizes normally used as beams.

$$M_{ux(equiv)} = M_{ux} + P_u\left(\frac{d}{2}\right)$$

Where:

$M_{ux(equiv)}$ = equivalent design bending moment for trial section, ft-K

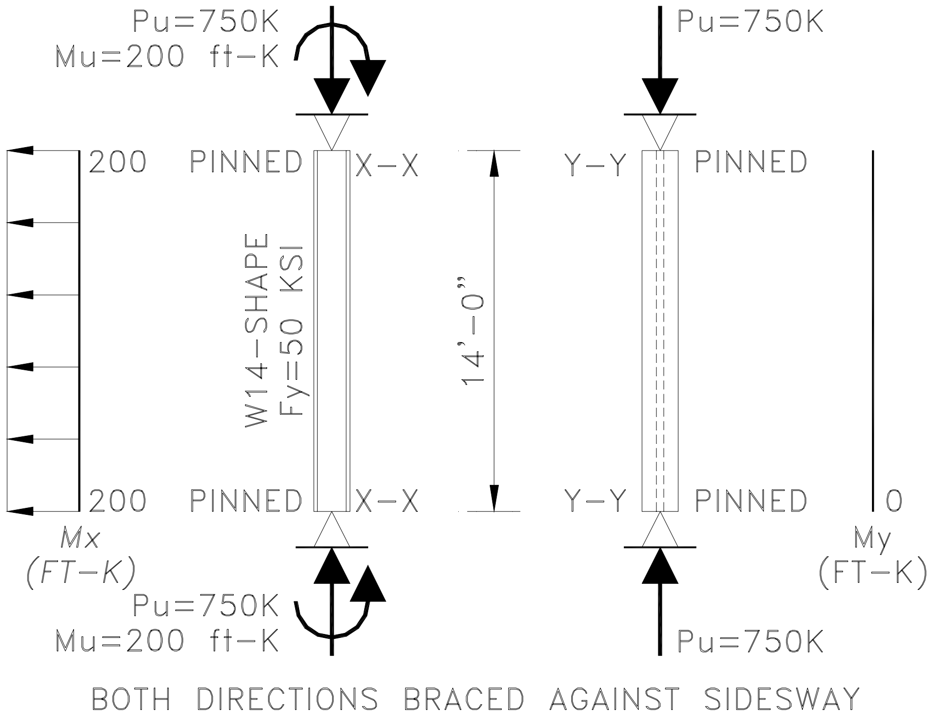
RMD DESIGN PROCEDURE

The design of a beam-column is accomplished by analysis of a trial section.

1) Compute required strength for trial section.
   - P_u {A4}
   - M_{ux} {A4}
   - M_{uy} {A4}
2) Select trial section Using AISC, Jura, or Alternate Trial Selection Method.
3) Compute design strength for trial section.
   - $\phi_c P_n$ {E2}
   - $\phi_b M_{nx}$ {F1}
   - $\phi_b M_{ny}$ {F1}
4) Compute second order effects for trial section.
   - M_{ux} {C1}
   - M_{uy} {C1}
5) Compute $\dfrac{P_u}{\phi P_n}$ and determine appropriate interaction equation for trial section.
6) Check appropriate interaction equation for trial section.
7) If appropriate interaction equation is satisfied, the trial section is acceptable. If not, select new trial section and return to Step 2.

EXAMPLE PROBLEM BMCOL #1

GIVEN:



REQUIRED: Design beam column

SOLUTION:

1) Jura trial selection

$$P_{u(equiv)} = P_u + M_{ux}\left(\frac{2}{d}\right) + M_{uy}\left(\frac{7.5}{b_f}\right) = 750\,K + \frac{(200\,ft-K)(12\,in/ft)2}{14\,in} = 1093\,K$$

$(KL)_x = (KL)_y = 14\,ft$ <u>y-axis governs</u>

Try W14x109: $\phi_c P_n = 1170\,K$ {Manual 4-23}

2) W14x109 – design strength – flexural yielding – x-axis

$L_p = 13.2\,ft$ {Manual 5-46}

$L_r = 43.2\,ft$ {Manual 5-46}

$L_p < L_b = 14\,ft < L_r$ <u>inelastic buckling</u>

$\phi_b M_n = \phi_b M_p = 720\,ft-K$ {Manual 5-46}

3) W14x109 – design strength – lateral torsional buckling – x-axis

$C_b = 1.0$ for constant moment diagram

$$\phi_b M_n = C_b (\phi_b M_n)_{chart} = 1.0(717\ ft - K) = 717\ ft - K \qquad \text{\{Manual 5-88\}}$$

4) W14x109 – design strength – flexure – x-axis

lateral torsional buckling limit state governs

$$\phi_b M_n = 717\ ft - K$$

5) W14x109 – second-order effects – x-axis

$$M_{nt} = 200\, ft - K$$

$$M_{lt} = 0\, ft - K$$

$$(KL)_x = 1.0(14\ ft)(12\, in/ft) = 168\, in$$

$$P_{e1} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29000\, ksi)(1240\, in^4)}{(168\, in)^2} = 12575\, K$$

No loading between supports, bent in single curvature

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{200\ ft - K}{200\ ft - K}\right) = 1.00 \qquad \text{\{Eq. C1-3\}}$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{1.00}{1 - \frac{750\, K}{12575\, K}} = 1.06 \quad \underline{\text{governs}} \qquad \text{\{Eq. C1-2\}}$$

$$B_{1(\min)} = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{lt} = 1.06(200\ ft - K) + B_2(0\ ft - K) = 212\ ft - K \qquad \text{\{Eq. C1-1\}}$$

6) Check interaction

$$\frac{P_u}{\phi P_n} = \frac{750\, K}{1170\, K} = 0.64 \geq 0.20 \quad \underline{\text{Use Eq. H1-1a}}$$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{ux}} + \frac{M_{uy}}{\phi_b M_{uy}}\right) = \frac{750\, K}{11706\, K} + \frac{8}{9}\left(\frac{212\ ft - K}{7170\ ft - K} + 0\right) \qquad \text{\{Eq. H1-1a\}}$$

$$0.64 + 0.26 + 0.00 = 0.90 < 1.00 \quad \underline{\text{ok}}$$

Check to see if next lightest section will work.

7) W14x99 - design strength - compression

$\phi_c P_n = 1060$ {Manual 4-23}

8) W14x99 - design strength – flexural yielding - x-axis

$L_p = 13.5\ ft$ {Manual 5-46}

$L_r = 40.6\ ft$ {Manual 5-46}

$L_p < L_b = 14\ ft < L_r$ <u>inelastic buckling</u>

$\phi_b M_n = 649\ ft - K$ {Manual 5-88}

9) W14x99 – design strength – lateral torsional buckling – x-axis

C_b = 1.00 for constant moment diagram

$\phi_b M_n = C_b (\phi_b M_n)_{chart} = 1.0(643\ ft - K) = 643\ ft - K$ {Manual 5-89}

10) W14x99 – design strength – flexure – x-axis

lateral torsional buckling limit state governs flexure

$\phi_b M_n = 643\ ft - K$

11) W14x99 - second order effects - x-axis

$$P_{e1} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29000\,ksi)(1110\,in^4)}{(168\,in)^2} = 11256\,K$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{1.00}{1 - \frac{750\,K}{11256\,K}} = 1.07 \quad \underline{\text{governs}}$$ {Eq. C1-2}

$B_{1(\min)} = 1.0$

$$M_u = B_1 M_{nt} + B_2 M_{lt} = 1.07(200\ ft - K) + B_2(0\ ft - K) = 214\ ft - K$$ {Eq. C1-1}

12) W14x99 - check interaction

$$\frac{P_u}{\phi P_n} = \frac{750\,K}{1060\,K} = 0.71 \geq 0.20$$ <u>Use Eq. H1-1a</u>

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{750\,K}{1060\,K} + \frac{8}{9}\left(\frac{214\ ft - K}{643\ ft - K} + 0\right)$$ {Eq. H1-1a}

$$= 0.708 + 0.296 + 0 = 1.00$$ <u>ok</u>

Select W14x99

BOLTED CONNECTIONS

TYPES OF BOLTS

In the early days of construction, structural steel connections were made with rivets. Installation of rivets was a time-consuming and labor intensive process. Eventually riveted connections were replaced by bolted connections, which were easier to install.

Common Bolts

ASTM A307 bolts, known as common bolts. The bolts are tightened using long-handled manual wrenches. The induced tension in the bolts is small and unpredictable. The bolts are satisfactory in building frames not subject to shock and vibration and in lightly loaded applications.

They are furnished in two grades:

- Grade A for general purpose
- Grade C for rods
- F_y is not specified {Manual 2-26}
- F_u = 58 or 60 ksi, depending on grade {Manual 2-26}

High Strength Bolts

Most bolted connections today consist of high strength bolts, either ASTM A325 or ASTM A490. Because they are made of high-strength steel, fewer bolts are needed for the same connection loads. The bolts can be tightened to large tensions, which produce high clamping forces between the connected parts.

- F_y is not specified
- $F_{u(A325)}$ = 105 or 120 ksi, depending on grade {Manual 2-26}
- $F_{u(A490)}$ = 125 ksi {Manual 2-26}
- Available in diameter up to 1½ inches
- Available as headed bolts only

SNUG-TIGHT AND FULLY TENSIONED BOLTS

Manual Page 16.1-51. J1.11. Limitations On Bolted And Welded Connections: For column splices, and connections subject to dynamic loads, fully tensioned high strength bolts (or welds) must be used.

In other cases connections may be made with A307 bolts or with high strength bolts that are installed snug tight.

Manual Page 16.1-58. Section J3.1 High Strength Bolts: Use of high strength bolts shall conform to the provisions of the *LRFD Specification For Structural Joints Using ASTM A325 or A490 Bolts*.

All ASTM A325 and A490 bolts shall be installed as fully tensioned, unless noted otherwise.

All other bolts need only be tightened to a snug tight condition, defined as the “tightness attained by either a few impacts of an impact wrench or the full effort of a worker with an ordinary spud wrench that brings the plies into full contact”.

METHODS FOR FULLY TENSIONING HIGH STRENGTH BOLTS

Manual Page 16.4-0. *Specification For Structural Joints Using ASTM A325 Or A490 Bolts: It* is published by the Research Council On Structural Connections (RCSC). It covers the design and installation of high strength bolts.

Manual Page 16.4-46. Section 8. Installation: Several methods are prescribed for tightening of high strength bolts in slip-critical connections and connections subject to direct tension.

- Turn-of-Nut Pretensioning {Page 16.4-48}
- Calibrated Wrench Pretensioning {Page 16.4-49}
- Twist-Off-Type Tension Control Bolt Pretensioning {Page 16.4-50}
- Direct Tension Indicator Pretensioning {Page 16.4-51}

SIZES OF BOLT HOLES

Manual Page 16.1-59. Section J3.2. Size And Use Of Holes: The maximum size of bolt holes shall be as given in Table J3.3, except for column base plates. From an engineering point-of-view, bolt holes should be as small as possible. From a construction point-of-view, bolt holes should be as large as possible.

Manual Page 16.1-62. Table J3.3. Nominal Hole Dimensions: Many AISC design and installation requirements in the AISC Specification and the Bolt Specification are tied to these definitions of nominal hole dimensions

- Notice that the standard hole is the bolt diameter plus $1/16$ inch. This is the most common hole size. Fabrication shops and AISC design aids are standardized to this hole size.
- Oversize and slotted holes are used to facilitate construction when additional "slop" is required to allow bolts to be installed.

SPACING AND EDGE DISTANCE OF BOLTS

AISC is concerned that the failure mode of bolted connection is by bearing between the bolt shank and the connecting plates. It is necessary to put limitation on the center to center spacing of bolts in the direction of force to preclude shear failure along the plate before bearing failure is achieved.

Manual Page 16.1-60. Section J3.3. Minimum Spacing: Defines the minimum bolt spacing requirements.

- Minimum bolts spacing = $2\frac{2}{3}d$
- Preferred bolt spacing = 3d
- Industry standard bolt spacing = 3"

Manual Page 16.1-60. Section J3.4. Minimum Edge Distance: Table J3.4 defines the minimum edge distance. It is necessary to put limitations on **minimum** edge distances between the bolt holes and the sides of the plate to preclude failure by tear out of the plate before bearing failure is achieved.

Manual Page 16.1-63. Table J3.4. Minimum Edge Distances: Indicates minimum edge distance from center of standard holes to edge of connecting part.

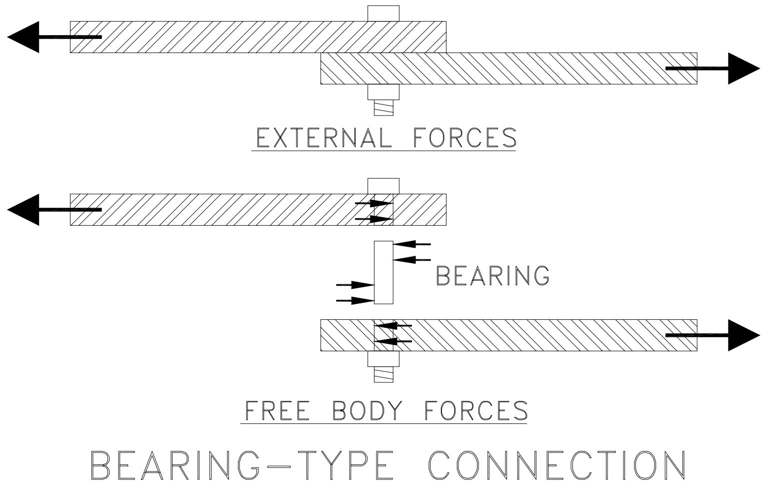
Manual Page 16.1-60. Section J3.5. Maximum Spacing And Edge Distance: **Maximum** spacing and edge distance requirements are intended to minimize the opportunity for corrosion by precluding gaps between the plies of connected parts.

BEARING-TYPE CONNECTIONS

In a bearing-type connection, there is slippage under the design load conditions.

The bolts come in contact with the connecting plates, resulting in bearing stresses.

The shank of the bolt fails in shear parallel to the line of force and perpendicular to the bolt axis.

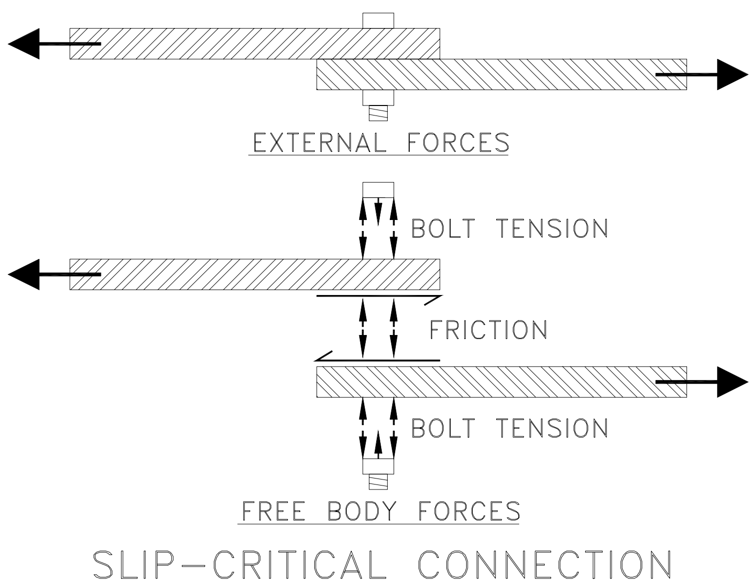


Bearing-type connections are installed with the bolts tightened to a snug tight condition.

SLIP-CRITICAL TYPE CONNECTIONS

In a slip-critical connection, the nut of the bolt is tightened so much that the resulting tension force on the bolt produces a sufficient clamping force to prevent slippage within the connection.

The bolt shank does not need to come in contact with the connection plates, hence, no bearing stresses between the shank and the bearing plates are produced.



To qualify as a slip-critical connection, the high-strength bolts must be installed as fully tensioned. This specified pretension mobilizes a significant frictional force between the two plates.

DESIGN STRENGTH

Manual Page 16.1-49. Section J1.1. Design Basis: Connection components shall be proportioned so that the design strength equals or exceeds the required strength.

$$V_u \text{ or } P_u \leq \phi R_n \qquad \text{\{A5.3\}}$$

Manual Page 16.1-50. Section J1.7. Minimum Strength Of Connections: Connections shall have a design strength of 10K, except for lacing, sag rods, and girts.

ASD Sec. J1.6: Connections carrying calculated stresses, except for lacing, sag bars and girts, shall be designed to support not less than 6 kips.

Bolt Areas

AISC and Structural Engineers use the gross bolt area: $A_b = \dfrac{\pi d^2}{4}$

Mechanical Engineers use the net tensile area: $A_b = \dfrac{\pi\left(d - \dfrac{0.9743}{n}\right)^2}{4}$

Where:

A_b = nominal unthreaded bolt body area, in^2
d = nominal bolt diameter, inch
n = threads per inch, unitless

Tension Strength Limit State

The shank of the bolt fails in tension parallel to the line of force and bolt axis.

Manual Page 16.1-62. Section J3.6. Design Tension Or Shear Strength: Applicable for bearing-type and slip-critical bolts. Design tension strength equals:

$$\phi R_n = \phi F_t A_b$$

Manual Page 16.1-61. Table J3.2. Design Strength Of Fasteners: Resistance factor and tensile strength of fasteners is listed as a function of fastener ASTM specification and location of threads relative to shear plane.

ASD Table J3.2, Allowable Stress On Fasteners

Shear Strength Limit State – Bearing-Type Connections

The shank of the bolt fails in shear parallel to the line of force and perpendicular to the bolt axis.

Manual Page 16.1-62. Section J3.6. Design Tension Or Shear Strength: Applicable for bearing-type bolts only. Design shear strength equals:

$$\phi R_n = \phi F_v A_b$$

Manual Page 16.1-61. Table J3.2. Design Strength Of Fasteners: Resistance factor and shear strength of bearing-type fasteners is listed as a function of fastener ASTM specification and location of threads relative to shear plane.

ASD Table J3.2, Allowable Stress On Fasteners

Shear Resistance Limit State – Slip-Critical Type Connections

The frictional resistance between the connected plies slips before the bolt can fail in shear or the connected part fails in bearing or excessive deformation.

Even though AISC regards a slip-critical connection as a serviceability limit state evaluated with service loads, they provide provisions with factored loads, as if it was a strength limit state.

Manual Page 16.1-64. Section J3.8a. Slip-Critical Connections Designed At Factored Load: Applicable for slip-critical connections only. Design shear resistance equals ϕR_{str} .

$$R_{str} = 1.13 \mu T_b N_s \qquad \text{\{Eq. J3-1\}}$$

Where:

ϕ = resistance factor
= 1.0 for standard holes
= 0.85 for oversized and short-slotted holes
= 0.70 for long-slotted holes transverse to the load direction
= 0.60 for long-slotted holes parallel to the load direction

R_{str} = design slip resistance per bolt, Kips

μ = mean slip coefficient, unitless
= 0.33 for Class A surfaces (unpainted mill scale or Class A coating)
= 0.50 for Class B surfaces (unpainted blast cleaned or Class B coating)
= 0.40 for Class C surfaces (galvanized or roughened surface)

T_b = minimum fastener tension given in Table J3.1, Kips

N_s = number of slip planes, unitless

ASD Table J3.2, Allowable Stress On Fasteners

Bearing Strength Limit State

The connected part fails by deformation at the bolt hole. Failure may either by "bearing" at the bolt/part interface or by excessive deformation of the bolt hole.

Manual Page 16.1-66. Section J3.10. Bearing Strength At Bolt Holes: Applicable for both bearing-type and slip-critical bolts. Design bearing strength equals ϕR_n.

Where:

ϕ = resistance factor = 0.75

(a) For bolts in connections with standard, oversized, and short-slotted holes or long-slotted holes with the slot parallel to the direction of bearing force:

When bolt hole deformation at service load is a design consideration:

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \quad \text{\{Eq. J3-2a\}}$$

$$\boxed{F_p = 1.2 F_u} \quad \boxed{\text{ASD Eq. J3-1}}$$

Where:

l_c = clear distance in direction of force from the edge of the bolt hole under consideration to the edge of either the next hole or the material, inch

t = material thickness, inch

When bolt hole deformation at service load is a not design consideration:

$$R_n = 1.5 l_c t F_u \leq 3.0 d t F_u \quad \text{\{Eq. J3-2b\}}$$

$$\boxed{F_p = 1.5 F_u} \quad \boxed{\text{ASD Eq. J3-4}}$$

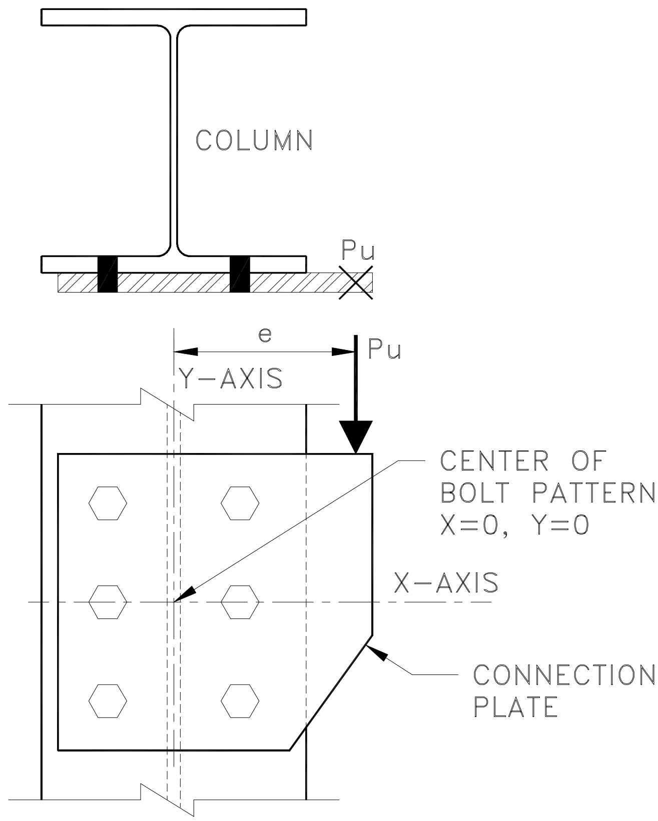
(b) For bolts in connections with long-slotted holes with the slot perpendicular to the direction of bearing force:

$$R_n = 1.0 l_c t F_u \leq 2.0 d t F_u \quad \text{\{Eq. J3-2c\}}$$

$$\boxed{F_p = 1.0 F_u} \quad \boxed{\text{ASD Eq. J3-2}}$$

COMBINED SHEAR AND TORSION (ECCENTRIC SHEAR)

When a load *P* passes through a line of action that does not pass through the center of the bolt group, the connection must be designed for the direct shear load *P* as well as the torsion load *P* times *e*. This situation is also called eccentric shear.



ECCENTRIC SHEAR CONNECTION

Manual Page 16.1-51. Section J1.8. Placement of Welds And Bolts: The connection center of gravity must coincide with the member center of gravity, unless the eccentricity is accounted for. Based on common practice, eccentricities can be neglected for statically loaded angles and tee-sections. This does not mean this is a good practice. Notice that wind and earthquake are dynamic loads, not static.

Manual Page 16.1-68. Section J5.1. Eccentric Connections: The centers of gravity of axially loaded members should converge on one point, unless the eccentricity is accounted for.

Manual Page 7-44. Table 7-18. Coefficient C For Eccentrically Loaded Bolt Groups: AISC has precalculated "C" values for a number of bolt group geometries. Variables considered are:

- number and spacing of bolt group rows
- number and spacing of bolt group columns
- load eccentricity from bolt group center
- angle of load relative to vertical

COMBINED SHEAR AND TENSION

Bearing-Type Connections

Manual Page 16.1-63. Section J3.7. Combined Tension And Shear In Bearing-Type Connections: Design tension strength equals $\phi F_t A_b$.

Where:

F_t = nominal bolt tensile strength as a function of the required factored load shear stress equations from Table J3.5, ksi

Manual Page 16.1-65. Table J3.5. Nominal Tensile Stress Limit (F_t), KSI, Fasteners In Bearing-Type Connections: The shear stress to tensile stress interaction is defined based on observed empirical relationships.

$$f_v = \frac{V_u}{A_b}$$

A307 bolts:	$F_t = 59 - 2.5 f_v \le 45\,ksi$	$\boxed{ASD: F_t = 26 - 1.8 f_v \le 20\,ksi}$
A325 bolts, threads inc.:	$F_t = 117 - 2.5 f_v \le 90\,ksi$	$\boxed{ASD: F_t = \sqrt{(44)^2 - 4.39 f_v{}^2}}$
A325 bolts, threads exc.:	$F_t = 117 - 2.0 f_v \le 90\,ksi$	$\boxed{ASD: F_t = \sqrt{(44)^2 - 2.15 f_v{}^2}}$

Where:

f_v = required factored load shear stress, ksi

V_u = required shear strength, Kips

Slip-Critical Connections

Manual Page 16.1-66. Section J3.9a. Slip-critical Connections Designed At Factored Loads: Design shear strength equals:

$$\phi R_{str} = \phi 1.13 \mu T_b N_s \left(1 - \frac{T_u}{1.13 T_b N_b} \right)$$

> ASD Sec. J3.6: … the maximum shear stress allowed by Table J3.2 shall be multiplied by the reduction factor $1 - f_t A_b / T_b$ …

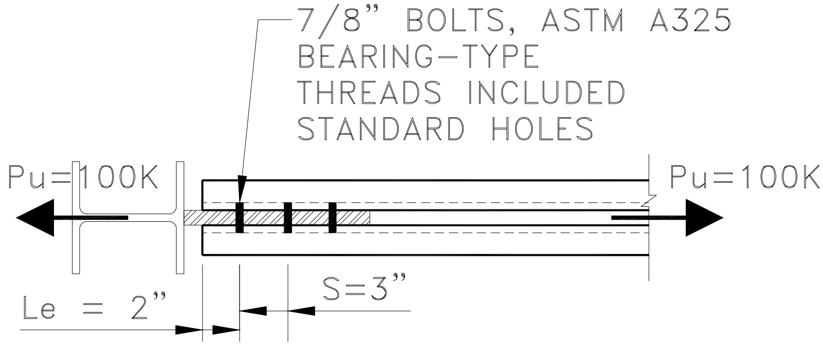
The shear strength for a slip-critical connection is the product of the normal force and an appropriate coefficient of friction. When there is applied tension to such a joint, the available normal force and shear strength is proportionately reduced.

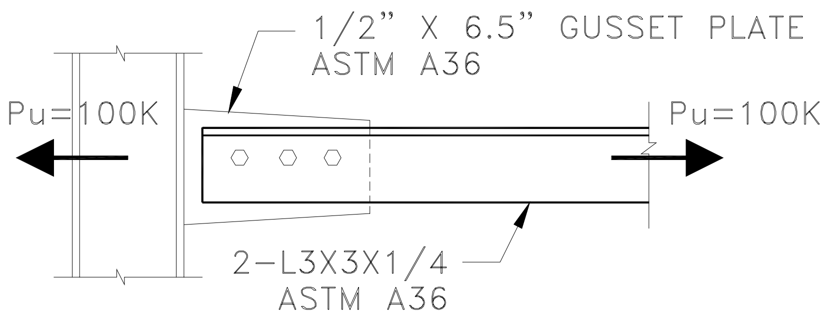
Manual Page 16.1-60. Table J3.1. Minimum Bolt PreTension, Kips:

> ASD Table J3.7, Minimum Pretension for Fully-tightened Bolts

EXAMPLE PROBLEM BOLTED #1

GIVEN:





REQUIRED: Determine the number of bolts required. Assume connecting parts are adequate. Neglect block shear rupture.

SOLUTION:

1) Design strength - shear

$$\phi R_n = \phi F_v A_b = 0.75(48\,ksi)(2)(0.601\,in^2) = 43.3\,K/bolt$$

$$\phi R_n = 2(21.6\,K) = 43.2\,K/bolt$$ {Manual 7-33}

2) Design strength - bearing

$$\phi R_n = \phi 1.2 l_c t F_u \le 2.4 d t F_u$$ {Eq. J3-2a}

$$= 0.75(1.2)(2\,in)(2)(0.25\,in)(58\,ksi) \le 0.75(2.4)(0.875\,in)(2)(0.25\,in)(58\,ksi)$$

$$= 52.2\,K \le 45.7\,K$$

$$\phi R_n = (91.3\,K/in)(2)(0.25\,in) = 45.65\,K/bolt$$ {Manual 7-34}

3) Bolt selection

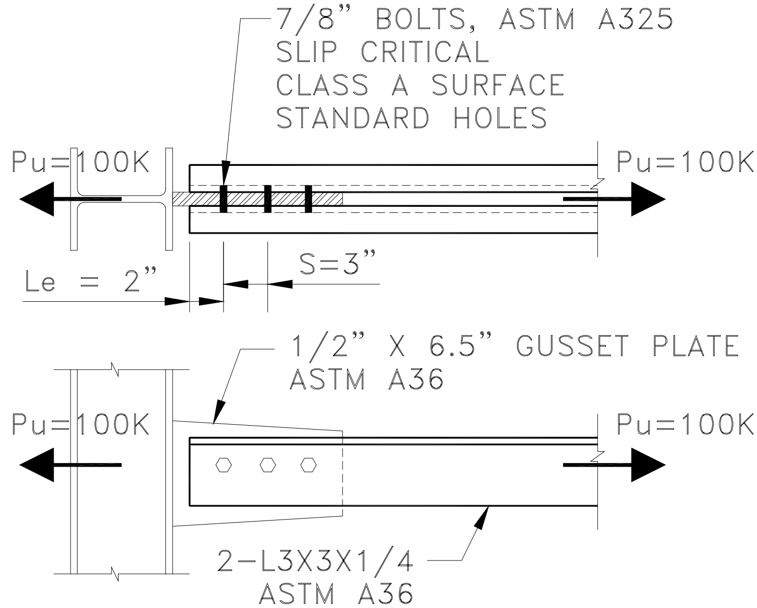
$\phi R_n = 43.3\,K/bolt$ governed by shear

$\#_{bolts} = \dfrac{100\,K}{43.3\,K/bolt} = 2.31$ say 3 bolts

Select: 3 Bolts

EXAMPLE PROBLEM BOLTED #2

GIVEN:



REQUIRED: Determine the number of bolts required. Assume connecting parts are adequate.

SOLUTION:

1) Design strength – shear

$T_b = 39\,K$ {Table J3.1}

$\phi R_{str} = \phi 1.13 \mu T_b N_s = 1.0(1.13)(0.33)(39\,K)(2) = 29.1\,K/bolt$

$\phi R_{str} = 29.1\,K/bolt$ {Manual 7-36}

2) Design strength – bearing

$\phi R_n = \phi 1.2 l_c t F_u \leq 2.4 d t F_u$ {Eq. J3-2a}

$= 0.75(1.2)(2\,in)(2)(0.25\,in)(58\,ksi) \leq 0.75(2.4)(0.875\,in)(2)(0.25\,in)(58\,ksi)$

$= 52.2\,K \leq 45.7\,K$

$\phi R_n = (91.3\,K/in)(2)(0.25\,in) = 45.65\,K/bolt$ {Manual 7-34}

3) Bolt selection

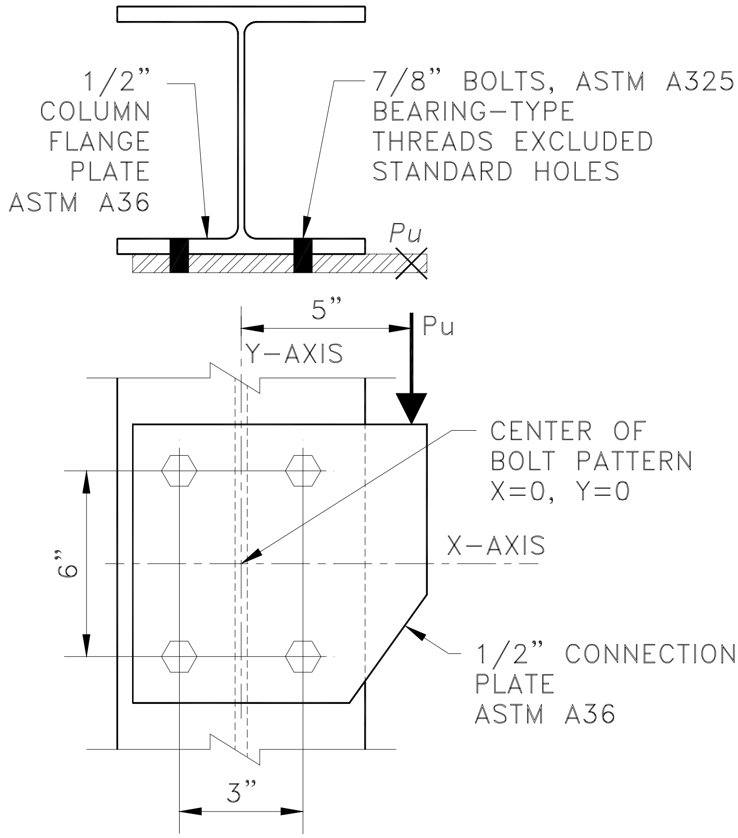
$\phi R_n = 29.1\,K/bolt$ <u>governed by shear</u>

$\#_{bolts} = \dfrac{100\,K}{29.1\,K/bolt} = 3.43$ say 4 bolts

Select: 4 Bolts

EXAMPLE PROBLEM BOLTED #3

GIVEN:



REQUIRED: Determine maximum bolt group design strength using LRFD Manual tables

SOLUTION:

1) Coefficient

n = 2, *s* = 6", *e* = 5", column spacing = 3"

$C = 2.10$ {Manual 7-44}

2) Bolt design strength - shear

27.1 K/bolt <u>previously determined</u>

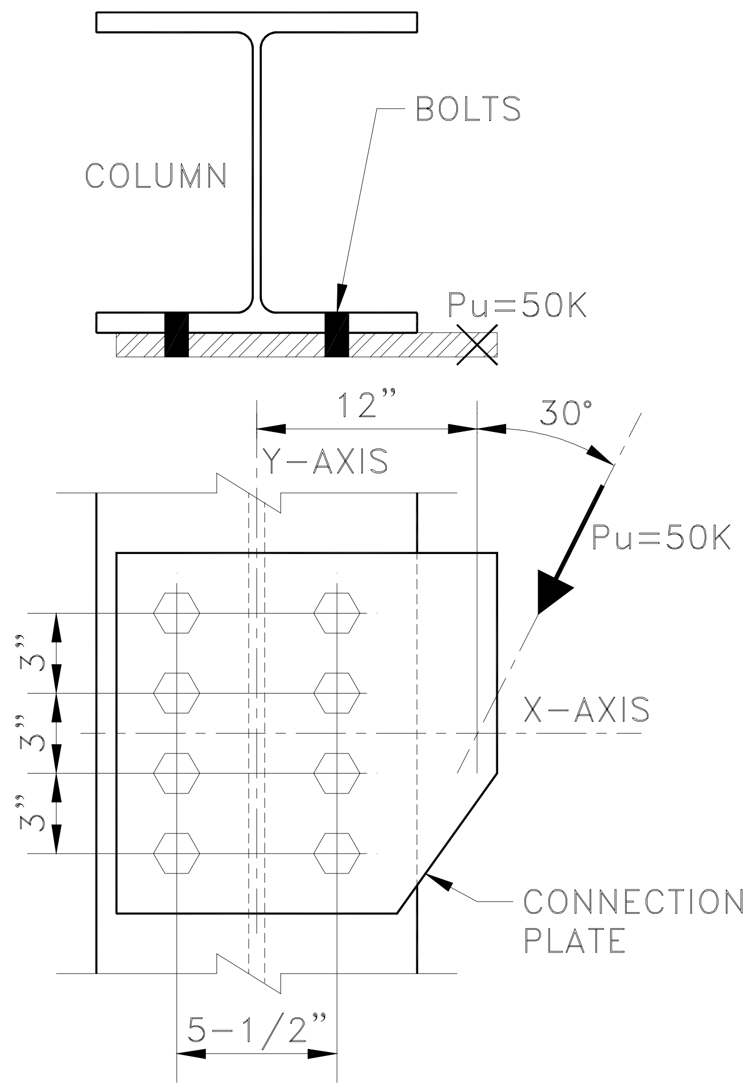
3) Group design strength

$C\phi r_n = 2.10(27.1K) = 56.9\,K$

Bolt Group Design Strength = 56.9 K

EXAMPLE PROBLEM BOLTED #4

GIVEN:



REQUIRED: Determine maximum required bolt design strength using the LRFD Manual tables

SOLUTION:

1) Coefficient

n = 4, s = 3", e = 12", ϕ = 30°, column spacing = 5.5"

C = 2.65 {Manual 7-52}

2) Bolt design strength - required

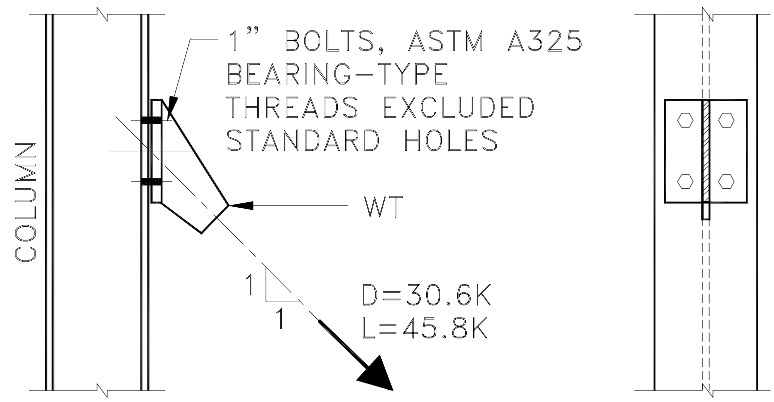
$$\phi r_n = \frac{P_u}{C} = \frac{50K}{2.65} = 18.9K$$

Required Bolt Design Strength = 18.9K

EXAMPLE PROBLEM BOLTED #5

GIVEN:

REQUIRED: Check adequacy of bolts. Assume connected parts are adequate.



SOLUTION:

1) Required strength

$$P_u = 1.4(30.6\,K) = 42.8\,K$$

$$P_u = 1.2(30.6K) + 1.6(45.8\,K) = 110\,K$$ governs

$$V_u = T_u = \frac{1}{\sqrt{2}}\left(\frac{110\,K}{4}\right) = 19.4\,K$$

2) Bolt design strength – shear

$$\phi R_n = \phi F_v A_b = 0.75(60\,ksi)(0.785\,in^2) = 35.3\,K > 19.4\,K = V_u$$ ok

$$\phi R_n = 35.3\,K$$ {Manual 7-33}

3) Bolt design strength- bearing

no information given, assume ok

4) Bolt design strength - tension

$$f_v = \frac{V_u}{A_b} = \frac{19.4\,K}{0.785\,in^2} = 24.7\,ksi$$

$$F_t = 117 - 2.0(24.7\,ksi) = 67.6\,ksi$$ {Table J3.5}

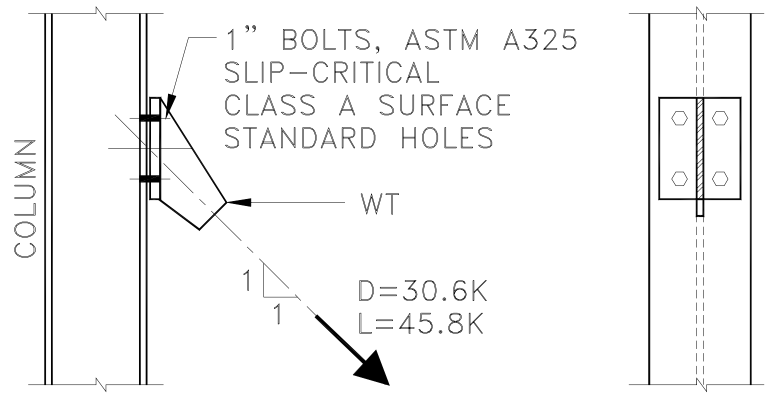
$$\phi R_n = \phi F_t A_b = 0.75(80\,ksi)(0.785\,in^2) = 47.1\,K > 19.4\,K = T_u$$ ok

Connection Is Acceptable

EXAMPLE PROBLEM BOLTED #6

GIVEN:

REQUIRED: Check adequacy of bolts. Assume connected parts are adequate.



SOLUTION:

1) Required strength (factored loads)

$$P_u = 1.4(30.6\,K) = 42.8\,K$$

$$P_u = 1.2(30.6K) + 1.6(45.8\,K) = 110\,K$$ governs

$$V_u = T_u = \frac{1}{\sqrt{2}}\left(\frac{110\,K}{4}\right) = 19.4\,K$$

2) Bolt design strength – tension

$$\phi R_n = \phi F_t A_b = 0.75(90\,ksi)(0.785\,in^2) = 53.0\,K > 19.4\,K = T_u$$ ok

$$\phi R_n = 53.0\,K$$ {Manual 7-35}

3) Bolt design strength- bearing

no information given, assume ok

4) Bolt design strength – shear (reduced applied by tension)

$$T_b = 51K$$ {Table J3.1}

$$\phi R_{str} = \phi 1.13 \mu T_b N_s\left(1 - \frac{T_u}{1.13 T_b N_b}\right) = (1.0)(1.13)(51K)(1)\left(1 - \frac{19.4\,K}{1.13(51\,K)(1)}\right)$$

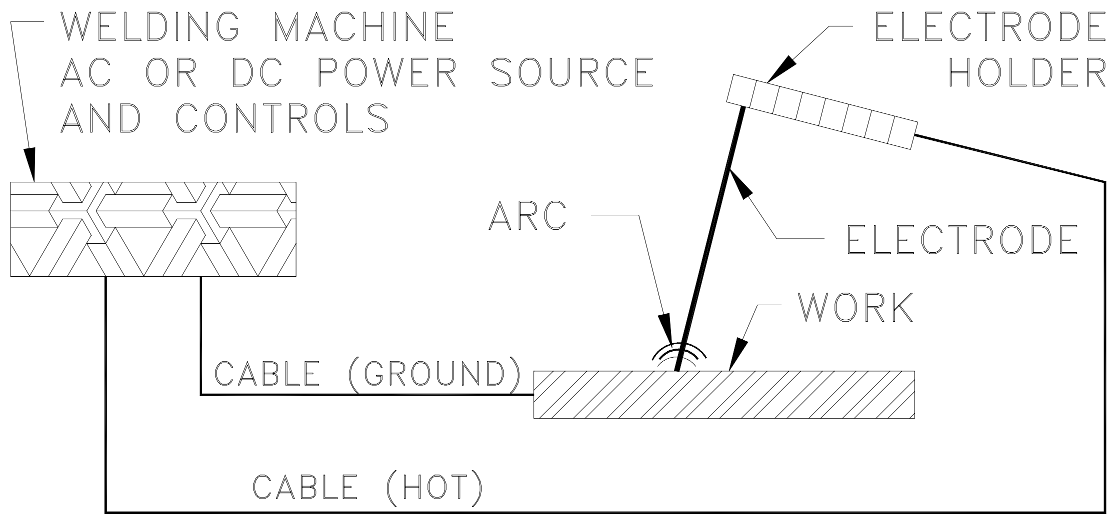
$$= (19.0\,K)(0.66) = 12.5\,K < 19.4\,K = V_u$$ ng

Connection Is Not Acceptable

WELDED CONNECTIONS

Welding is the localized joining of metals by heating the base materials to above their melting points, with or without the addition of filler metals.

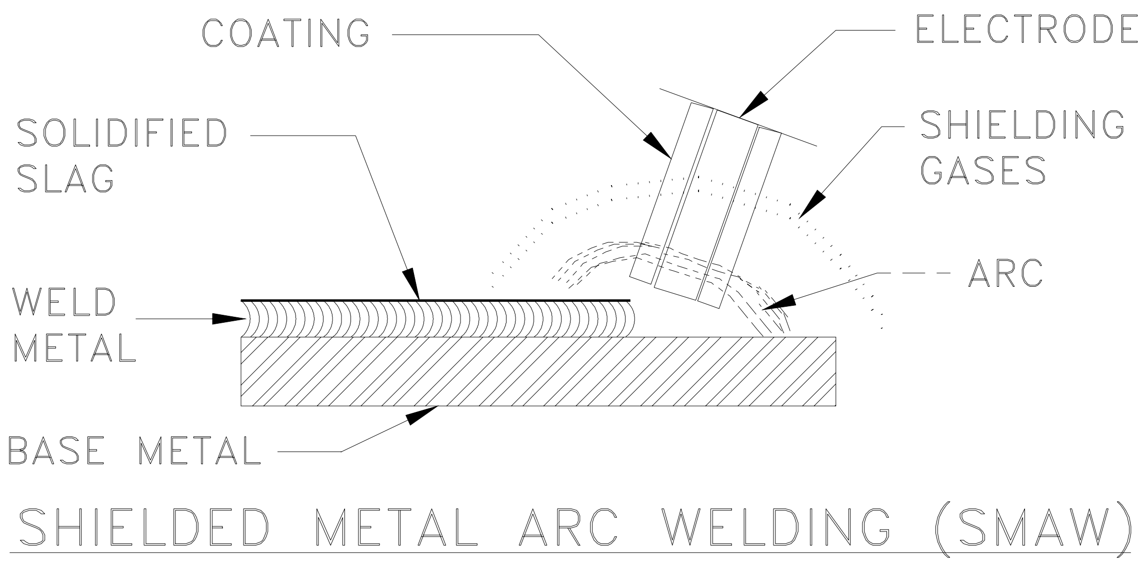
STRUCTURAL ARC WELDING



BASIC ARC WELDING CIRCUIT

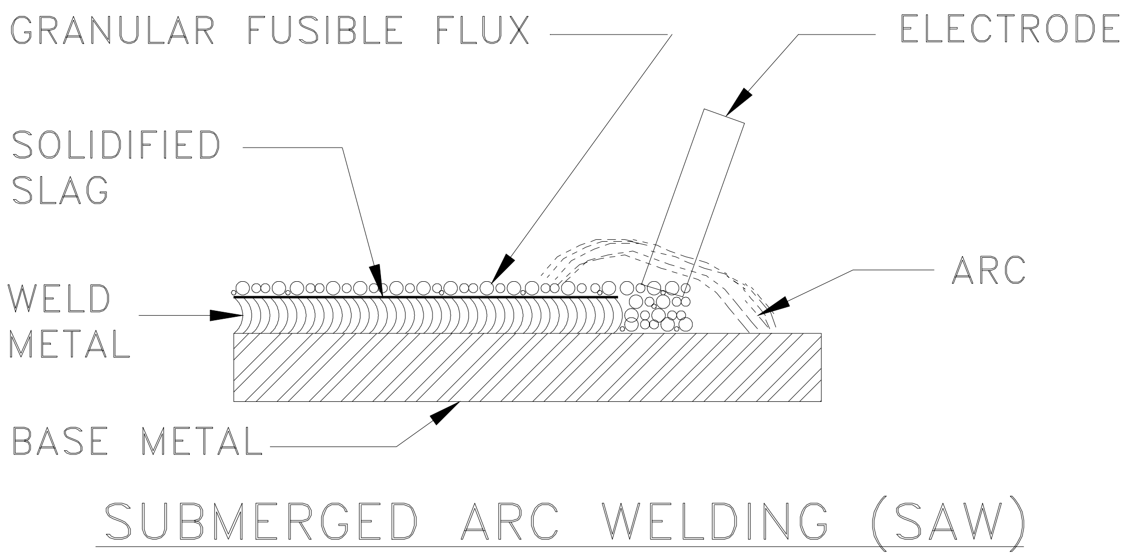
Almost all structural welding is arc welding. The power source is connected by a ground cable to the work piece. The power source is also connected by a "hot" cable to an electrode. When the circuit is energized and the electrode tip touches the work piece, the circuit is completed. When the electrode tip is then withdrawn from, but held close to the work piece, an arc is created across the gap. The arc produces a temperature of about 6500°F, which melts the base metal, and any filler metal. After the melted metals cool and solidify, a solid piece of bonded metal is left, the completed weld. The pool of molten metal can hold a fairly large amount of gases in solution. If the pool is not shielded from the surrounding atmosphere, it will chemically combine with the free oxygen and nitrogen. A relatively brittle, nonductile weld would result. As a result, arc welds are typically shielded by using either a specially coated electrode, or a granular flux.

SHIELDED METAL ARC WELDING (SMAW)



Shielded Metal Arc Welding (SMAW) is commonly called stick welding or manual welding. It is the most commonly used process because it is simple and versatile. Welds can be made in all positions and in many difficult-to-reach areas. The coated electrode is consumed as it is transferred to the base metal during the welding process. The electrode coating produces a gaseous shield to exclude air and stabilize the arc. The electrode coating introduces materials to refine the grain structure of the metal. The electrode coating produces a blanket of slag over the molten metal and solidified weld. The slag protects the weld from nitrogen and oxygen that would otherwise react with the hot metals. The slag also serves to slow down the cooling process of the weld, reducing potential brittleness. The work area must be kept dry to preclude the introduction of hydrogen and oxygen into the molten material. Similarly, wind speeds must be fairly low to preclude dissipation of the protective shielding gasses. Constant replacement of new electrodes for the consumed electrodes decreases the time actually spent welding and adds to the labor cost.

SUBMERGED ARC WELDING (SAW)



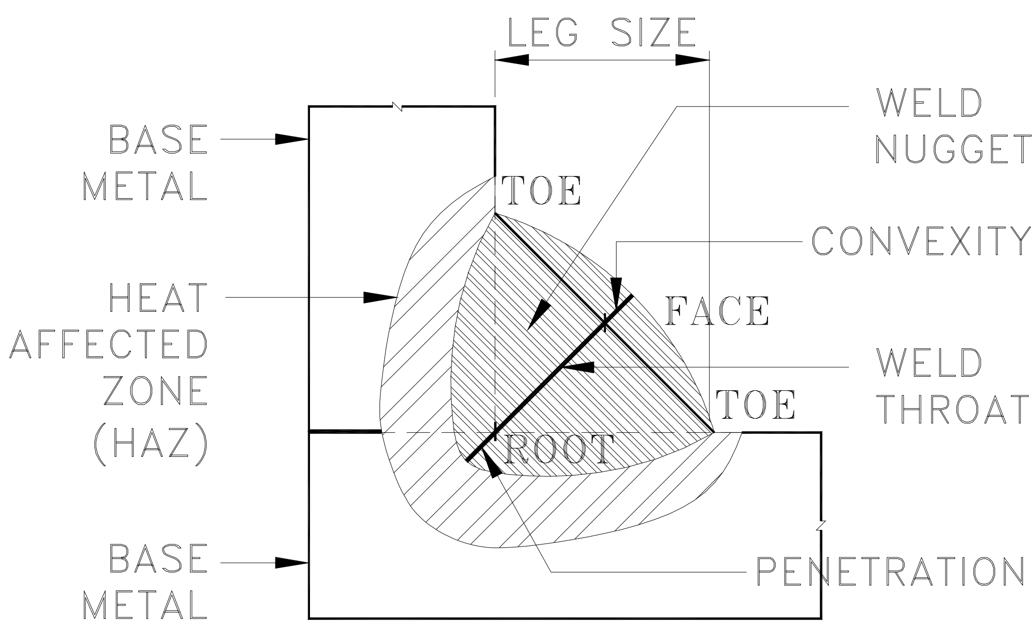
The arc and the weld metal are buried in a layer of flux, which protects the weld metal from contamination and concentrates the heat into the base metal. The base metal (work piece) must be laid flat to allow gravity to hold the granular flux in place. The molten flux arises through the weld pool, deoxidizing and cleaning the weld metal, and forms a protective slag over the newly deposited metal. The flux is a granular, fusible material. Submerged Arc Welding (SAW) is commonly automated and is used in fabrication shops because of economies.

- Fast deposition of weld metal
- More uniform and better quality welds
- Good ductility and impact resistance
- Higher heat input, deeper penetration into base metal

STUD WELDING

Stud welding involves the same basic principles and metallurgical aspects as any other arc welding procedure, in that a controlled electric arc is used to melt the end of the stud or electrode and a portion of the base metal. The stud is plunged automatically into the molten metal and a high quality fusion weld is accomplished where the weld is stronger than the stud itself. Stud welding is applicable to mild steel, stainless steel and aluminum. Studs may be fed to the welding gun manually or automatically, depending upon the application. In either case, a simple squeeze of the welding gun trigger produces a positive attachment between the weld stud and the base metal in a split second.

FILLET WELDS



FILLET WELD TERMINOLOGY

Base Metal is the work pieces being joined by the weld.

Weld Nugget is the melted filler metal and base metal joining the work pieces.

Heat Affected Zone (HAZ) is the base metal whose mechanical properties or microstructure have been altered by the heat of welding and subsequent cooling.

- Stronger, but more brittle than other base metal
- Susceptible to absorption of hydrogen
- Preheat of base metal will slow down HAZ cooling rate, reduces cooling rate and allows absorbed hydrogen to escape

Weld Throat is the weld metal area used in strength calculations.

Convexity is weld metal area not used in strength calculations.

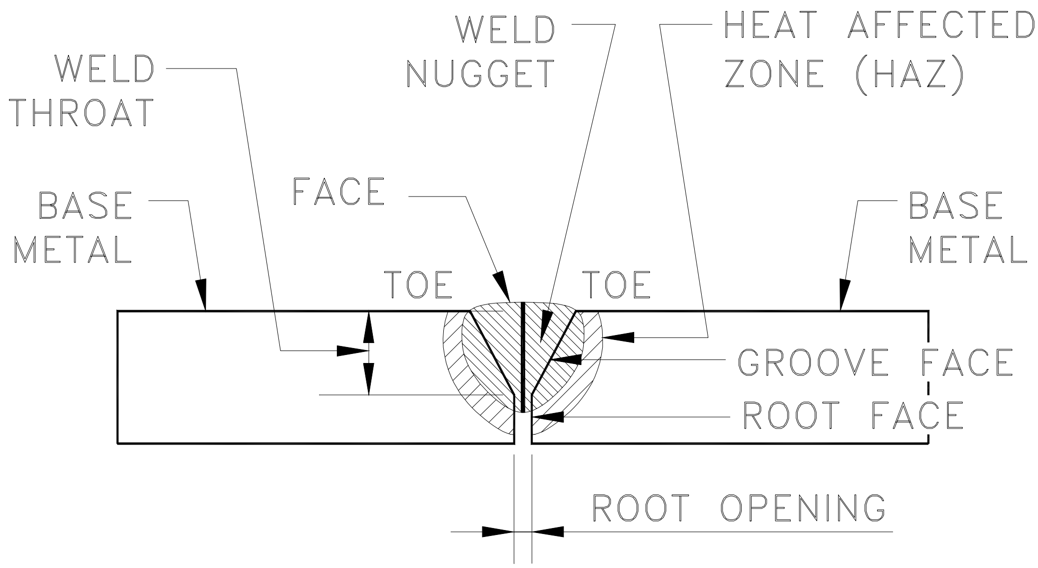
Penetration is weld metal not used in strength calculations.

Leg Size is the dimension specified on design drawings.

Face is the exposed weld surface on the side from which the welding was done.

Toe is the junction of the weld face and base metal.

GROOVE WELDS



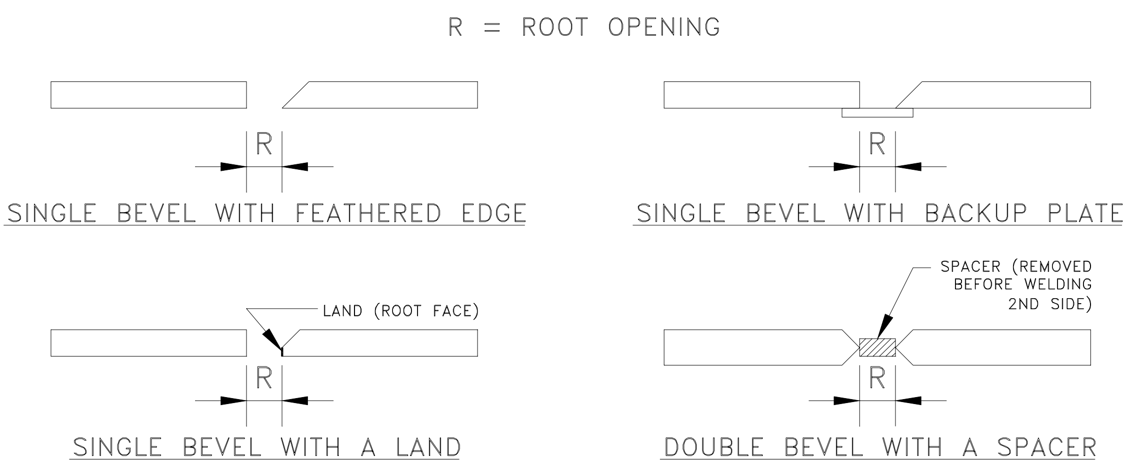
GROOVE WELD TERMINOLOGY

Root Opening is the separation of the base metal at the root.

Groove Face is the base metal included in the groove.

Root Face is the base metal included in the root.

Edge Preparation is the preparation of the base metal by cutting, cleaning, plating, or other means.



GROOVE WELD EDGE PREPARATION

DESIGN STRENGTH

Manual Page 16.1-7. Section A5. Design Basis: Connection components shall be proportioned so that the design strength equals or exceeds the required strength.

$$V_u \ or\ P_u \le \phi R_n \qquad \{A5.3\}$$

Manual Page 16.1-50. Section J1.7. Minimum Strength Of Connections: Connections shall have a design strength of 10K, except for lacing, sag rods, and girts.

ASD Sec. J1.6: Connections carrying calculated stresses, except for lacing, sag bars and girts, shall be designed to support not less than 6 kips.

Manual Page 16.1-51. Section J1.11. Limitations On Bolted And Welded Connections: For column splices and connections subjected to dynamic loads, welds or fully tensioned high strength bolts must be used.

Manual Page 16.1-56. Section J2. 4. Design Strength: The design strength shall be the lower of:

$$\phi R_n = \phi F_{BM} A_{BM}$$

or

$$\phi R_n = \phi F_W A_W$$

Where:

- ϕ = resistance factor as defined in Table J2.5
- R_n = weld nominal strength, Kips
- F_{BM} = base metal nominal strength as defined in Table J2.5, ksi
- A_{BM} = base metal cross-sectional area, in^2
- F_w = weld electrode nominal strength as defined in Table J2.5, ksi
- A_w = weld effective area, in^2

Manual Page 16.1-57. Table J2.5. Design Strength Of Welds: Table lists resistance factors and nominal strengths for:

Types of weld

- complete-joint-penetration groove weld
- partial-joint-penetration groove weld
- fillet welds
- plug or slot welds

Types of stress

- Tension
- compression
- shear

Material

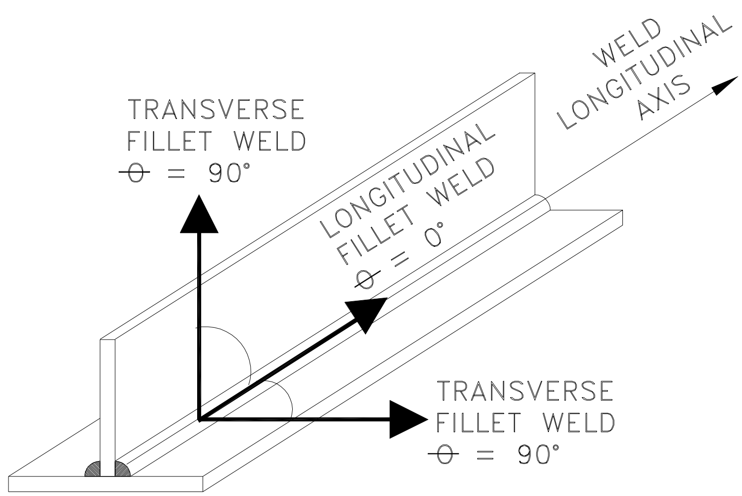
- base metal
- weld electrode

Footnote (g) refers to alternate design strength requirements in specification Appendix J2.4.

ASD Table J2.5, Allowable Stress On Welds

Manual Page 16.1-115. Appendix Section J2.4. Design Strength: As an alternate procedure, the design strength for fillet welds of linear weld groups loaded through the center of gravity may be determined by the following equations:

$$F_w = 0.60 F_{EXX}\left(1.0 + 0.5\sin^{1.5}\theta\right) \qquad \text{\{Eq. A-J2-1\}}$$



STRENGTH OF WELDS

Where:

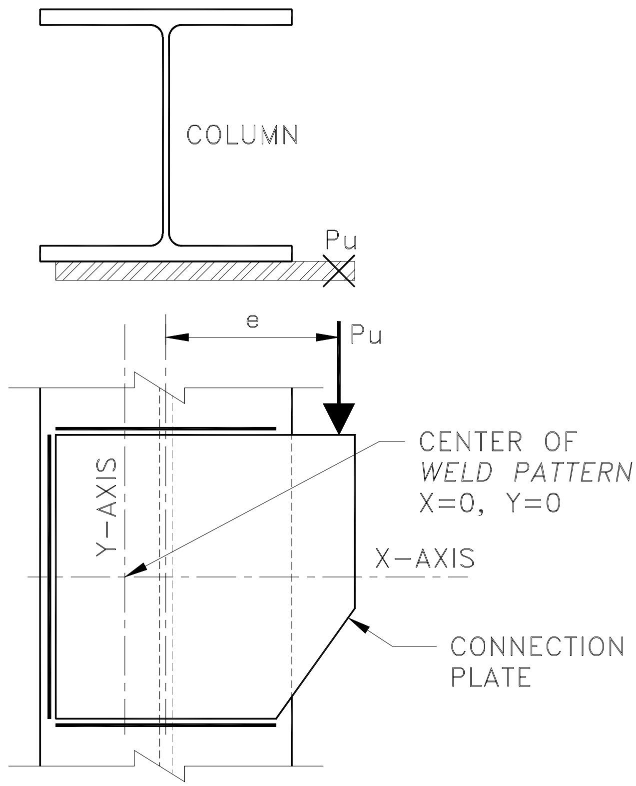
ϕ = resistance factor = 0.75

F_{EXX} = weld electrode minimum specified strength, ksi

θ = angle of loading measured from the weld longitudinal axis, degrees

IN-PLANE ECCENTRIC LOADS

When a load *P* passes through a line of action that does not pass through the center of the weld group, there will be an eccentric loading effect. The connection must be designed for the direct shear load *P* as well as the torsion load *P* times *e*. This situation is also called eccentric shear.



ECCENTRIC SHEAR CONNECTION

Specification

Manual Page 16.1-51. Section J1.8. Placement of Welds And Bolts: The connection center of gravity must coincide with the member center of gravity, unless the eccentricity is accounted for.

Based on common practice, eccentricities can be neglected for statically loaded angles and tee-sections. This does not mean this is a good practice. Notice that wind and earthquake are dynamic loads, not static.

Manual Page 16.1-68. Section J5.1. Eccentric Connections: The centers of gravity of axially loaded members should converge on one point, unless the eccentricity is accounted for.

OUT-OF-PLANE ECCENTRIC LOADS

Elastic Method

The same principles apply as with shear and torsion.

$$f_s = \frac{P_u}{A_e} = \frac{P_u}{TL}$$

$$S = \frac{bd^2}{6} = \frac{T(L)^2}{6}$$

$$f_b = \frac{M}{S} = \frac{P_u e}{\frac{T(L)^2}{6}} = \frac{6P_u e}{T(L)^2}$$

$$f_v = \sqrt{(f_s)^2 + (f_b)^2}$$

Where:

f_s = shear stress parallel to load due to direct shear, ksi
L = weld or plate length, in
t = effective weld throat or plate thickness, in
S = bending elastic section modulus, in^2
f_b = bending stress perpendicular to load due to eccentricity, ksi

Note that the maximum bending stress occurs at the farthest point from the neutral axis. The maximum shear stress occurs at the neutral axis. The nominal shear stresses are equal at all points. Combination of the maximum bending stress with the nominal shear stress is conservative.

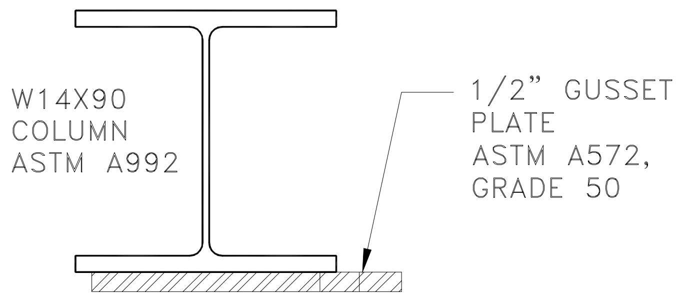
Strength Method

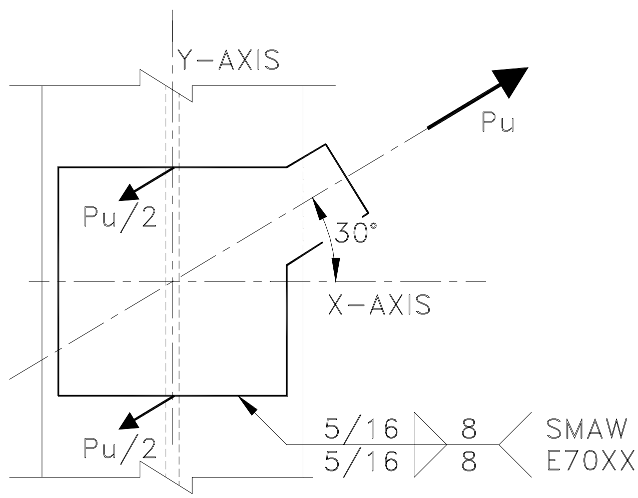
The same principles apply as with shear and torsion.

Manual Page 8-52. Table 8-5. Coefficient C For Eccentrically Loaded Weld Groups: This table addresses shear and bending with the special case where k = 0. Note that the solution will yield the weld size for each of the welds indicated. The total weld size would be 2D.

EXAMPLE PROBLEM WELDED #1

GIVEN:





REQUIRED: Determine weld design shear strength.

SOLUTION:

1) Fillet weld limitations

t_f = 0.710", thicker plate = 0.710"

$t_{min} = 1/4\, in$ {Table J2.4}

$t_{max} = 1/2\, in - 1/16\, in = 7/16\, in$ {J2.2b}

welds are within limitations <u>ok</u>

2) Design strength - SMAW weld shear

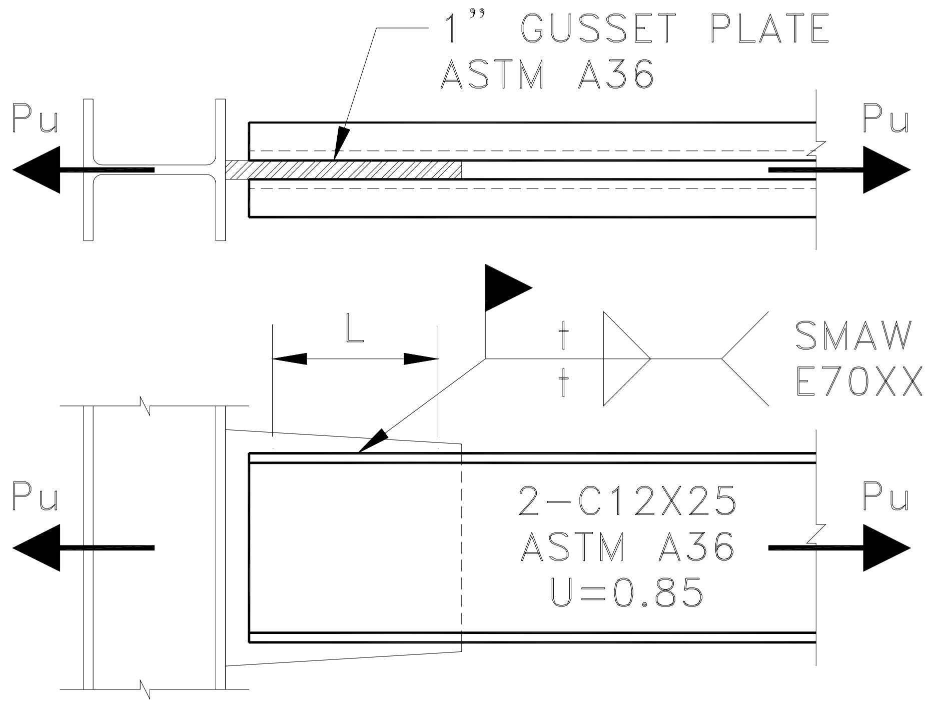
$F_w = 0.60 F_{EXX}\left[1 + 0.50 \sin^{1.5}\theta\right] = 0.60(70\, ksi)\left(1 + 0.50 \sin^{1.5} 30^o\right) = 49.4\, ksi$ {Eq. A-J2-1}

$\phi F_W A_w = \phi(49.5 ksi)(t_e L_e) = 0.75(49.5\, ksi)(2)(0.707)\left(5/16\, in\right)\left(8\, in/in\right) = 131\, Kips$

Design Strength = 131 K

EXAMPLE PROBLEM WELDED #2

GIVEN:



REQUIRED: Design weld to develop full design tensile capacity of channels.

SOLUTION:

1) Design strength - tension on gross section

$$A_g = 2\left(7.34\,in^2\right) = 14.7\,in^2$$

$$P_u \le \phi_t F_y A_g = 0.90\left(36\,ksi\right)\left(14.7\,in^2\right) = 476\,K$$

2) Design strength - tension on net section

$$A_e = UA_g = 0.85\left(14.7\,in^2\right) = 12.5\,in^2$$

$$P_u \le \phi_t F_u A_e = 0.75\left(58\,ksi\right)\left(12.5\,in^2\right) = 544\,K$$

3) Fillet weld limitations

t_w = 0.387"

thicker plate = 1"

$$t_{\min} = \frac{5}{16}in$$ {Table J2.4}

$$b_f = 2\frac{7}{8}in$$

$$t_{\max} = 2\frac{7}{8}in - \frac{1}{16}in = 2\frac{13}{16}in$$ {J2.2b}

for economy, try $t = t_{\min} = \frac{5}{16}in$

4) Design strength - SMAW weld shear

$$\phi F_W A_w = \phi(0.60F_{EXX})(t_e L_e) = 0.75(0.60)(70\,ksi)(4)(0.707)\left(\frac{5}{16}in\right)\left(L\frac{in}{in}\right) = 27.8L\,Kips$$

$$V_u = 476\,K = 27.8L\,Kips = \phi F_w A_w$$

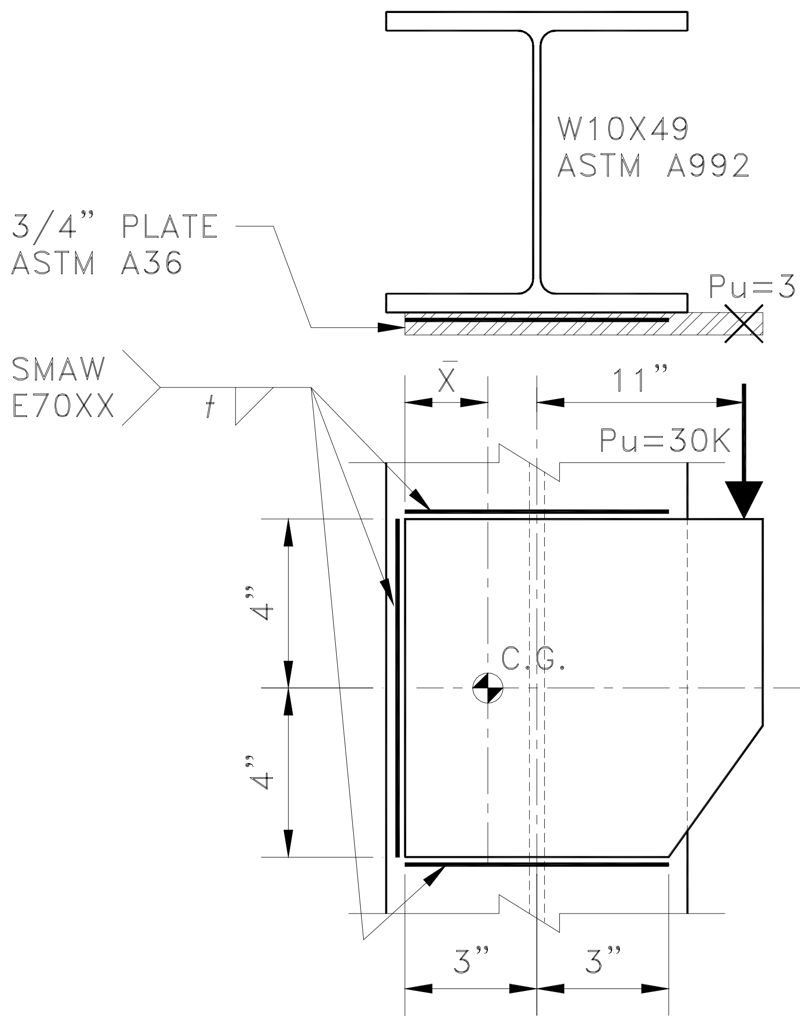
$$L_{req} = \frac{476\,K}{27.8\,K/in} = 17.1in$$ say 18 in

$$L_{\max} = 100\left(\frac{5}{16}in\right) = 31.2\,in$$ ok {Manual 16.1-55}

Select: 5/16" Fillet Welds, 18" Long

EXAMPLE PROBLEM WELDED #3

GIVEN:



REQUIRED: Design fillet welds using the Manual Tables

SOLUTION:

1) Fillet weld limitations

$t_{f(W10x49))} = 0.560\,in$

thicker plate = $\frac{3}{4}in$

$t_{\min} = \frac{1}{4}in$ {Table J2.4}

$t_{\max} = \frac{3}{4}in - \frac{1}{16}in = \frac{11}{16}in$ {J2.2b}

2) Weld group coefficients

C_1 = 1.00 for E70XX electrode {Manual 8-51}

$$k = \frac{6\,in}{8\,in} = 0.75$$

x = 0.225 by interpolation {Manual 8-76}

$$\bar{x} = 0.225(8\,in) = 1.80\,in$$

$$a = \frac{11in + 3\,in - 1.80\,in}{8\,in} = 1.52$$

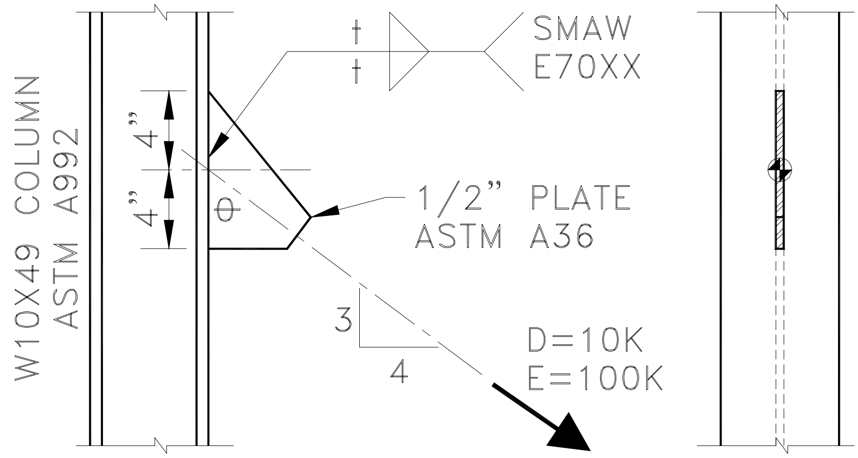
$C \approx$ 1.20 by interpolation {Manual 8-76}

$D_{req} = \dfrac{P_u}{CC_1L} = \dfrac{30\,K}{1.20(1.00)(8\,in)} = 3.13$ say $\dfrac{1}{4}in$, within limitations ok

Select: 1/4" Fillet Welds

EXAMPLE PROBLEM WELDED #4

GIVEN:



REQUIRED: Design fillet welds.

SOLUTION:

1) Required strength

$$P_u = 1.4(10\,K) = 14.0\,K$$

$$P_u = 1.2(10\,K) + 1.0(100\,K) = 112\,K$$ <u>governs</u>

2) Fillet weld limitations

$t_{f(W10x49)}$ = 0.560"

thicker plate = 0.560"

$$t_{\min} = \frac{1}{4}in$$ {Table J2.4}

no weld along edge of either plate, t_{max} is not applicable

3) Design strength - SMAW weld shear

$$f_{weld} = \frac{P_u}{L_e t_e} = \frac{112\,K}{(8\,in)2(0.707t)} = \frac{9.90}{t}ksi$$

$$\phi F_w = \phi(0.60 F_{EXX})(1 + 0.50 \sin^{1.5}\theta)$$ {Eq. A-J2-1}

$$= 0.75(0.60)(70\,ksi)\left(1.0 + 0.50\left(\frac{4}{5}\right)^{1.5}\right) = 42.8\,ksi = \frac{9.90}{t}ksi = f_{weld}$$

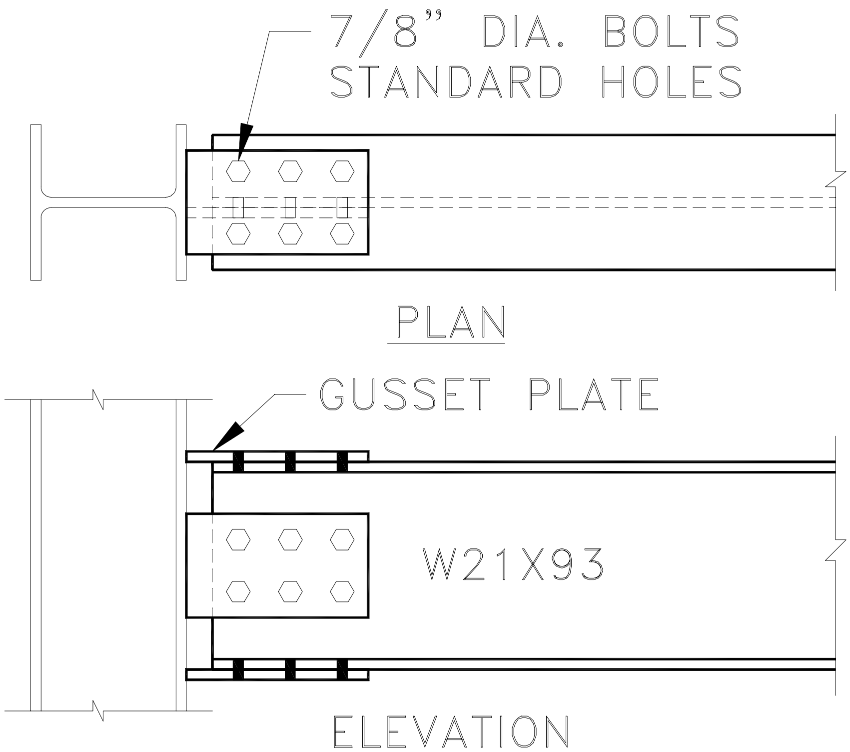
$$t_{req} = \frac{9.90\,K/in}{42.8\,ksi} = 0.231in$$ say $\frac{1}{4}in = t_{\min}$

Select 1/4" Weld Size

HOMEWORK

HOMEWORK PROBLEM #1

GIVEN:

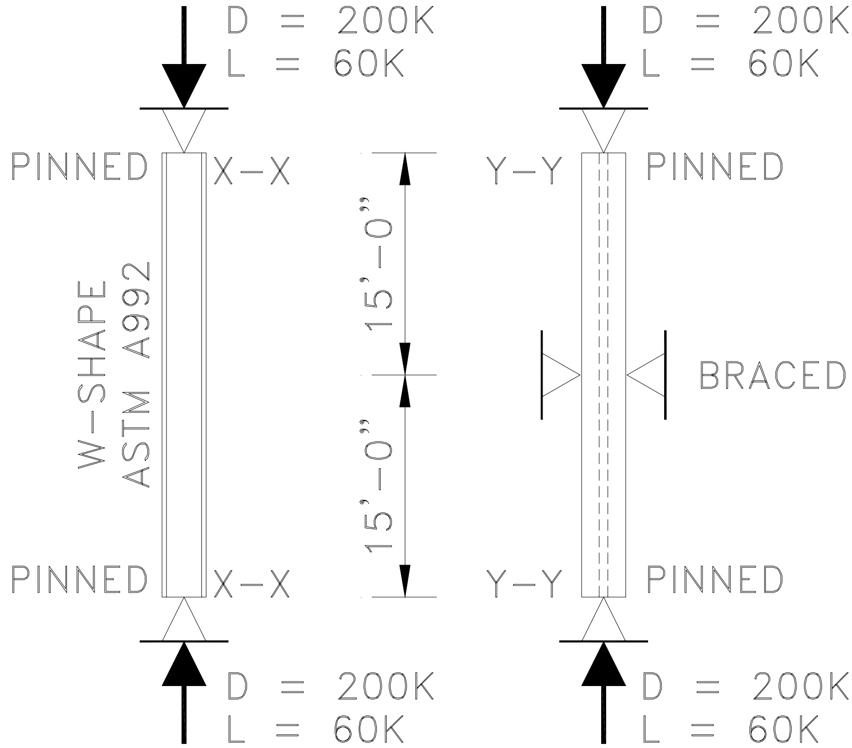


REQUIRED: Determine net area of beam.

SOLUTION:

HOMEWORK PROBLEM #2

GIVEN:

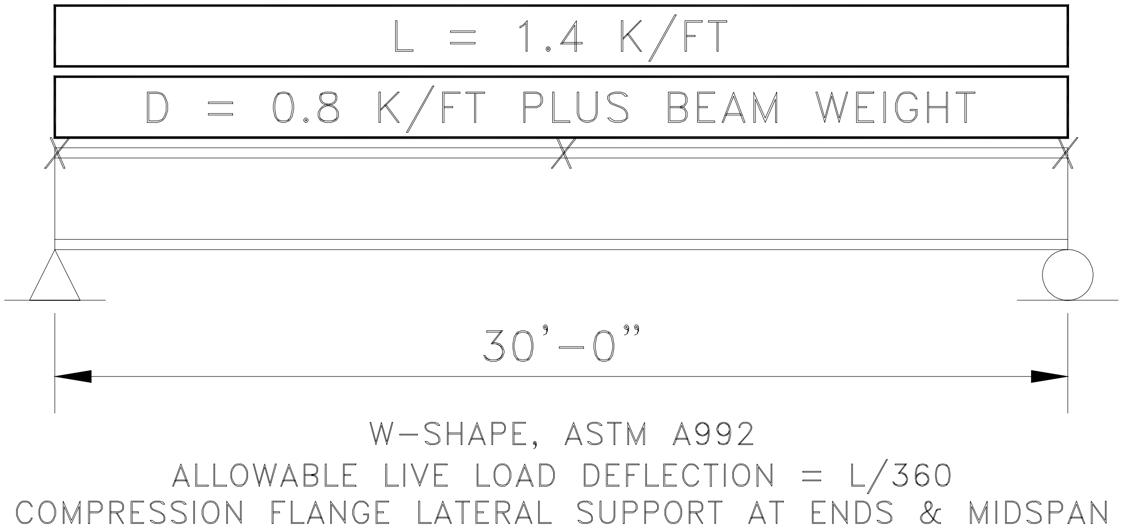


REQUIRED: Select lightest column.

SOLUTION:

HOMEWORK PROBLEM #3

GIVEN:

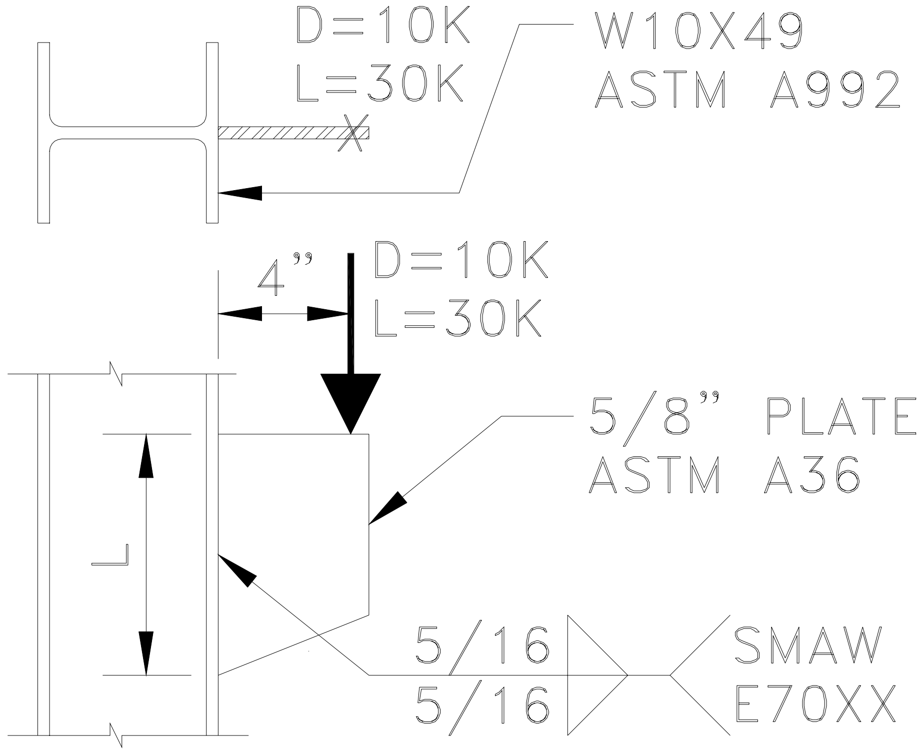


REQUIRED: Select lightest W-Shape. Consider flexure, shear, and deflection.

SOLUTION:

HOMEWORK PROBLEM #4

GIVEN:



REQUIRED: Design fillet welds.

SOLUTION: